The Impact of Office Dimension, Layout, Path Loss Exponent, and Multipath Fading on the k-Neighbors Connectivity Problem

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Abstract—Topology Control (TC) in ad-hoc networks, particularly Wireless Sensor Networks (WSN), aims to lower node energy consumption by limiting interference and collisions. A key issue regarding TC is to solve the k-neighbors connectivity problem, i.e., the minimal amount of neighbors needed to build a fully connected network. Usually, this problem has been studied based on flat, unrealistic scenarios or in an analytic way. In this work, we simulate some indoor scenarios taking into account the dimension, office layout, path loss exponent (PLE), and multipath fading to estimate the minimum k-neighbors that allows for the overall connectivity of a given WSN. Overall, our results showed that the obstacles interfered on the k values by increasing them. The path loss exponent is related to such k increases, especially when the offices have the same layout. However, for different layouts, we showed that k is not only driven by the PLE, but also by the layout. Moreover, multipath fading could help TC protocols to set k to smaller values. As a counterpart, multipath fading increased the energy consumption.

Index Terms—Computer network management; Distributed computing; Topology; Discrete event simulation; Electromagnetic propagation

I. INTRODUCTION

The challenges posed by Wireless Sensor Networks (WSN) have attracted the attention of several researchers in the last years, especially because WSNs establish new information collection and broadcast models, with huge potential for new applications such as environmental monitoring, “intelligent” buildings, and so on.

On the other hand, a main problem that WSNs pose is the limited battery capacity of their sensor nodes, which requires an effective Transmission Power Control (TPC) to be employed in each node in order to maintain a given connectivity degree [1]. However, minimizing the energy consumption and maintaining the connectivity are conflicting goals that influence other aspects of network operation, such as the quality of the signal received at the receiver, the range of a transmission, and the magnitude of the interference it creates to the other receivers, among others.

An example of technique to deal with TPC setup is Topology Control (TC). In theory, TC Protocols (TCP) need to be fully distributed and only local information can be used for decision-making in reducing traffic overhead [2]. Thus, such a Distributed TCP (DTCP) should [3] have the following characteristics: To be fully distributed and asynchronous; to be localized; to generate a topology that preserves the original network connectivity and relies, if possible, on bidirectional links; to generate a topology with small physical degree; and to rely on ’low-quality’ information. Although DTCPs can follow several approaches – for instance, they can be divided in Location-based TC; Direction-based TC; and Neighbor-based TC – they eventually rely on the nodes’ ability: (i) to determine the number and identity of neighbors within the maximum transmitting range; and (ii) to build an ordering of the neighboring set, e.g., based on link quality. In summary, DTCPs have to minimize the k-neighbors variable while keeping the overall connectivity.

To address this problem, we used Zerkalo, our propagation analysis tool introduced in a previous work [4], to simulate the deployment of a WSN in different indoor scenarios as representative examples of office buildings. By using Zerkalo, we could determine the minimum suitable k, for each scenario, that allowed the overall connectivity to be achieved considering the electromagnetic effects caused by propagation barriers and multipath fading. In overall, the main contributions of this work are: (a) to find the number k for several parameterized scenarios as a function of N (amount of nodes); (b) to analyze the impact of the dimensions, layout, path loss exponent (PLE)\(^1\), and the impact of

\(^1\)The traditional abbreviation of path loss exponent is n but we will adopt PLE to differentiate it from the variable that represents the amount of nodes
emitting electromagnetic phenomena on the \(k\)-neighbors problem within indoor parameterized scenarios.

The remainder of the paper is organized as follows. Section II briefly presents related work. In Section III, we introduce our simulation tool and the physical parameters assumed in the simulations. Section IV presents the experimental evaluation we conducted, including a description of the simulated indoor scenarios, the results we obtained, and a discussion related to the main findings that surprisingly contradicted the common sense in some cases. Finally, our conclusion and ongoing works are presented in Section V.

II. RELATED WORK

Topology Control is the art of coordinating nodes’ decisions regarding their transmitting ranges, in order to generate a network with the desired properties (e.g. connectivity while reducing node energy consumption and/or increasing network capacity [3]).

Links are formed by nodes choosing the power level at which they transmit. How many neighbors should each node be connected to achieve overall network connectivity? This question is known as the \(k\)-neighbors connectivity problem [3] and is formally defined as follows: given a set \(N\) of nodes, which is the minimum value of \(k\) such that the \(k\)-neighbors graph \(G_k\) built on \(N\) is strongly connected [3]?

Acting on the number \(k\) of neighbors to which every node is connected, it is possible to control the overall network connectivity. The higher the \(k\), the better the network connectivity. On the other hand, a small value of \(k\) is desirable for spatial reuse. Thus the optimal choice for \(k\) is to determine the minimum value such that the corresponding \(G_k\) graph is connected.

Works in the 1970s and 1980s suggested that the “magic number” of nearest neighbors should be six or eight [5]. Determining the \(k\) in theory is a very difficult problem, which has been partially solved only recently.

By assuming that \(n\) nodes are placed uniformly at random, Ref. [1] shows that \(k\) for overall connectivity is \(k = c \log n\) for some constant \(c\), with \(0.074 < c \leq 5.1774 + \epsilon\) where \(\epsilon\) is an arbitrary small positive value.

The \(k\)-neighbors problem was also investigated in [6], which simulated flat scenarios in details, obtaining values that were compatible with [1]. In particular, Table I summarizes their results for the minimum value of \(k\) that guaranteed a connected topology with a probability of at least 0.95.

However, we have not found works in the literature that analyzed the \(k\)-neighbors connectivity problem considering propagation barriers and multipath fading in a deterministic way.

<table>
<thead>
<tr>
<th>(N)</th>
<th>(k(P&gt;0.95))</th>
<th>(k(P=1))</th>
<th>(N)</th>
<th>(k(P&gt;0.95))</th>
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<td>7</td>
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<td>1000</td>
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</tr>
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</table>

TABLE I

CRITICAL NEIGHBOR NUMBER FOR DIFFERENT VALUES OF \(N\)

III. SIMULATION TOOL

In this section, we briefly explain Site Specific Propagation (SISP) Models and our tool, Zerkalo.

A. Site Specific Propagation Model

The simplest software approach for radio-wave propagation modeling at high frequencies (VHF to SHF) is semi-empirical, such as the well-known exponential path-loss model. Radiowave propagation models using detailed terrain databases are commonly referred SISP models. Smaller scenarios (usually indoors) may benefit from more complex and accurate approaches such as ray-tracing modeling. In this technique, the main propagation paths (rays) are deterministically found based on the common electromagnetic phenomena of reflection, refraction, and scattering, which includes diffraction. Ray-tracing is usually carried out two-fold, using either greedy methods or image theory [7]. With the ever growing available numerical capacity of computers, ray-tracing models have increasingly become more accurate as propagation prediction tools. Some researchers even expect that deterministic modeling may prevail in a near future, as the preferred approach for propagation prediction, even outdoors [8]. Among several SISP academic implementations, we can cite [9] and [10]. Recent analyzes about multipath fading in Ad-Hoc networks and WSN can be found in [11] and [12].

B. Zerkalo

In a previous work [4], we developed a SISP tool called Zerkalo (mirror in Russian) based on ray-tracing (image method) [7] that simulates the electromagnetic propagation in a given scenario and uses discrete event simulation techniques. Besides free-space propagation, Zerkalo also simulates the electromagnetic phenomena of reflection and refraction by computing the multipath interference due to reflections up to a desired order. Zerkalo’s algorithm complexity is \(O(n^r)\), where \(r\) is the reflection order and \(n\) is the number of obstacles.

In the design of Zerkalo, we assumed the so-called narrowband hypothesis that considers that the transmitted signal’s spectral content is narrow enough around the carrier (dozens or hundreds of KHz depending on the conditions) so that the technique fading can be considered flat [8]. The region most affected by this kind
of fading are those close to walls, specially the ones near to the corners [12].

We have also assumed the following test parameters: 0.122 m wavelength, half wave dipole antennas (1.64 dB gain) for transmission and reception [8], the capture threshold is 10 dB, and CSMA/CA-based MAC. We have modeled the error in the RSSI as 10% of receive power around the correct value. The receiver sensitivity and carrier sense threshold will vary according to the scenario. In all scenarios, the antennas’ heights were half way between floor and roof, such that the major propagation effects were concentrated on the horizontal plane comprising all antennas, simplifying the propagation problem to a 2D analysis.

For the analysis presented below (except in subsection IV-A2), we assumed a multipath fading threshold value of half the power received in the main propagation path that is usually the direct path. Specifically, if the (complex) sum of all multipath phasors is below half of the main component power (3 dB), then the signal will be codified, otherwise not.

The DTCP protocol chosen as reference is the XTC [13], which is suitable for static networks in most of the cases. It is based on the concept of ‘link quality’, so that we can consider in our simulation the multipath fading as a ‘link quality’ component, beyond the power receiver, and is Neighbor-Based.

IV. EXPERIMENTAL EVALUATION

In this section, we present the evaluated scenarios, computing the $k$ variable according to $N$ for each environment. Later on, we analyze and discuss the effect of dimension, layout, PLE, and multipath fading on results.

A. Scenarios

Beyond a flat, plan scenario, we choose four indoor scenarios, shown in Figure 1, for analyzing the $k$–neighbors problem, plus a flat scenario. Two are didactic and the remainings were taken for realistic scenarios extracted from [14] and [15], respectively. We conducted 200 random simulations for each pair $k \times N$, showing in the graphics the average result computed using the values collected in the cases where the connected nodes was greater than 95% and 99%, respectively, according to the parameters of the previous section. The results for all scenarios are shown in Figure 2, without and with multipath (except for the flat, where multipath is not applicable).

1) Flat Scenario: We choose a flat scenario of 45x45 m$^2$ in order to compare the results with scenarios with obstacles. It will be useful to see the differences that the obstacles may cause to the $k$-neighbors problem.
2) Tic-Tac-Toe Scenario: The didactic test scenario, shown in the top of Figure 1, is composed of 9 rooms, similarly arranged as in the Tic-Tac-Toe game. The rooms are apart by 15 cm wide walls with relative permittivity ($\varepsilon_r$) equal to 4.444 [8]. We used two different Tic-Tac-Toe scenarios in order to compare the dimension effect. The first one, called normal Tic-Tac-Toe (TTT), where each room is 15x15 m$^2$. The second one, called small Tic-Tac-Toe (sTTT), which has 5x5 m$^2$ rooms.

Figure 3 illustrates how Zerkalo computes path loss, with multipath interference and the radiation in the three rooms located at the center, the corner, and the lateral, respectively, taking the multipath fading threshold value equal to 3 dB. The darkest points in Figure 3 show the most affected points by the multipath interference, considering the transmitter relative position.

3) Kubisch Scenario: Kubisch scenario, shown in the middle of Figure 1, was taken out from [14]. The physical layout (18x7 m$^2$) consists of four rooms connected by a hallway as shown in the middle of Figure 1. In [14], the walls were assumed to be infinitesimally thin, constituting no obstacle for radio communication. Assuming a more realistic scenario, we consider the rooms are apart by wood divisions with $\varepsilon_r = 4.000$ [8].

4) Parallel Computing Laboratory (PCL): Our group, in described [15] the design of a new electronic device capable of measuring the energy cost of basic wireless communication events, including scan, connection, and data transport, illustrating the potentials of that equipments by fine measuring the energy costs of the Bluetooth protocol operation in practical situations. It is useful because it allows to compare the results obtained.
B. Analyze and Discussion of the results

In this section, we analyze the results from the previous subsection, decomposing same results of Figure 2 into Figure 4. Specifically, we will analyze: the impact of the dimension, the office layout and; and the multipath fading effect on the $k$-neighbors problem. The PLE of each scenario is shown in Table II.

1) Obstacles: As common sense predicts, according to Figure 2, we note that the obstacles causes the effect of increasing the $k$ value. In all scenarios, we have larger $k$ values than in the flat spaces. This result by itself justifies the study of this problem in a parametrized scenario.

2) Dimension: According to Figure 4(a), we notice that in scenarios with the same layout but with different dimension, the behavior of the graphics differs. Our results showed that bigger scenarios needs a smaller $k$ in order to keep the overall connectivity. In our case, the TTT scenario has a smaller PLE (3.68) than $s$TTT (3.86). It could induce us to infer that scenarios with smaller PLE will allow a smaller $k$ value, but, as we see in the following results, it does not occur always. We also made tests changing the scenarios material, and, as we suspect, $k$ grew when $\varepsilon_r$ increased.

3) Office Layout and PLE: The office layout produces important results, as we can see in Figure 4(b). Although Kubisch’s PLE is 3.08 and PCL’s PLE is 3.69, the $k$ value is almost the same. Yet, if we compute an weighted average ($\sum k_i N_i / \sum N_i$), we notice that PLC results is a bit smaller than Kubisch. Therefore, we can not make inference on $k$ based only on PLE. In spite of that, shows that we can not study this problem in generic simulators, such as ns-2, which does not considers scenario specific conditions and only allows to adjust the PLE.

4) Multipath Fading Effect: The multipath effect has two behaviors in WSNs. When we have few sensor nodes, this phenomenon has a prejudicial effect, forcing the nodes to increase the $k$ number. However, when the $N$ value increases, the difference between the two approaches (not considering and considering multipath) favors the scenario that considers such phenomenon. This occurs because the multipath effect obstructs links between nodes that are relatively close, obligating the sensor to apply a higher transmission power. The result is that the overall network is more connected, although wasting more energy. The multipath fading benefit effect is very clear in small Tic-Tac-Toe scenario, as we can see in Figure 4(c), a bit less, such as in PCL, or even equivalent as in Kubisch.

A possible explanation for these results can be understood through Figure 5. Suppose a scenario with 5 nodes and $k$ set to 3, where 4 nodes (A, B, C and D) are in a room and node E is outside of such room. According to the arrangement of Figure 5(a), node E will not take part of the overall network. On the other hand, it will take part of the network if $k$ would be set to 4, instead of 3. However, if we consider a multipath fading effect, the link between A and B could be broken, according to Figure 5(b). Considering that they need to be connected to 3 peers, A and B will increase their transmitting power from a real scenario with Zerkalo. Parallel Computing Laboratory (PCL) is a typical office (14.7x15 m²), divided by standard wood and glass divisions.

### Table II

<table>
<thead>
<tr>
<th>Scenario</th>
<th>PLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat</td>
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<tr>
<td>Tic-Tac-Toe (TTT)</td>
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</tr>
<tr>
<td>small Tic-Tac-Toe ($s$TTT)</td>
<td>3.86</td>
</tr>
<tr>
<td>Kubisch</td>
<td>3.08</td>
</tr>
<tr>
<td>PCL</td>
<td>3.69</td>
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</tbody>
</table>

### Fig. 4. Effect on Results
until contact E (wasting more energy), thus forming new links. In this example, considering multipath fading, $k=3$ is enough to make all network connected.

V. CONCLUSION

This work presented an in-depth analysis on the amount of neighbors to build a fully connected network within realistic indoor scenarios. We computed $k$ for five different parameterized scenarios and discussed the effect of office dimension, layout, and multipath fading on the $k$-neighbors problem.

Overall, our results showed that the obstacles interfered on the $k$ values by increasing them. The path loss exponent (PLE) is related to such $k$ increases, specially when the offices have the same layout. However, for different layouts, we showed that $k$ is not only driven by the PLE, but also by the layout. Moreover, multipath fading could help DTCP to set $k$ to smaller values. As a counterpart, multipath fading increased the energy consumption.

As further works, we plan to extend our analysis to mobile networks, in 3D scenarios, using different antenna heights, and to aggregate other radio irregularities besides multipath interference.

Fig. 5. Multipath Fading Affecting $k$

References