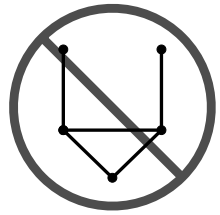
A Perfect Path from Computational Biology to Quantum Computing

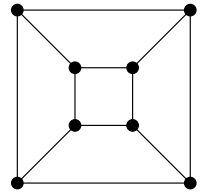
Celina Miraglia Herrera de Figueiredo

COPPE
UFRJ

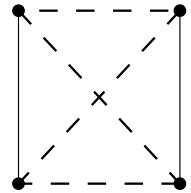
XI LAGOS 2023



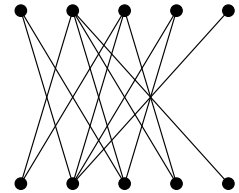
Celina, 1991



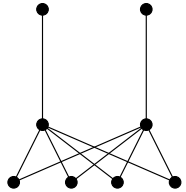
Luerbio, 1998



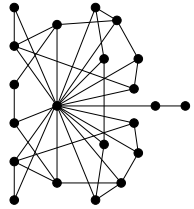
Simone, 2002



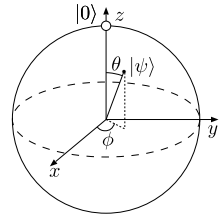
Vânia, 2004



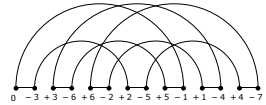
Cláudia, 2005



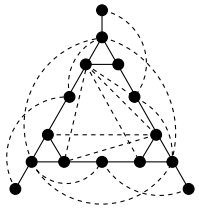
Vinicius, 2006



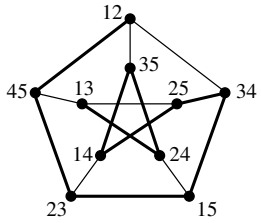
Luis Kowada, 2006



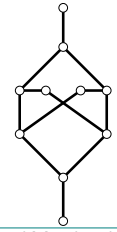
Rodrigo, 2007



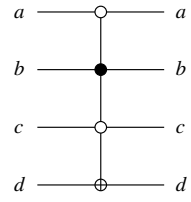
Rafael Bernardo, 2008



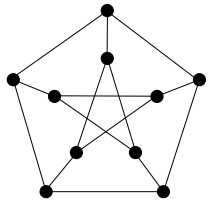
Letícia, 2009



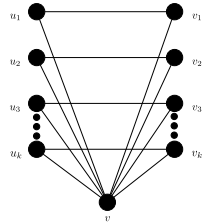
Raphael Machado, 2010



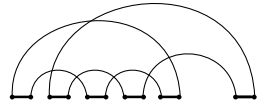
André, 2013



Diana, 2013

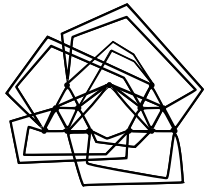


Hélio, 2014

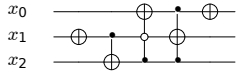


Luís Felipe, 2017

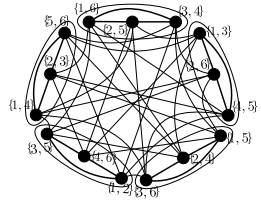
Ana Luísa, 2017



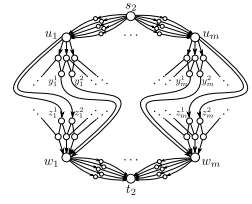
Alexandre, 2020



Edinelço, 2021

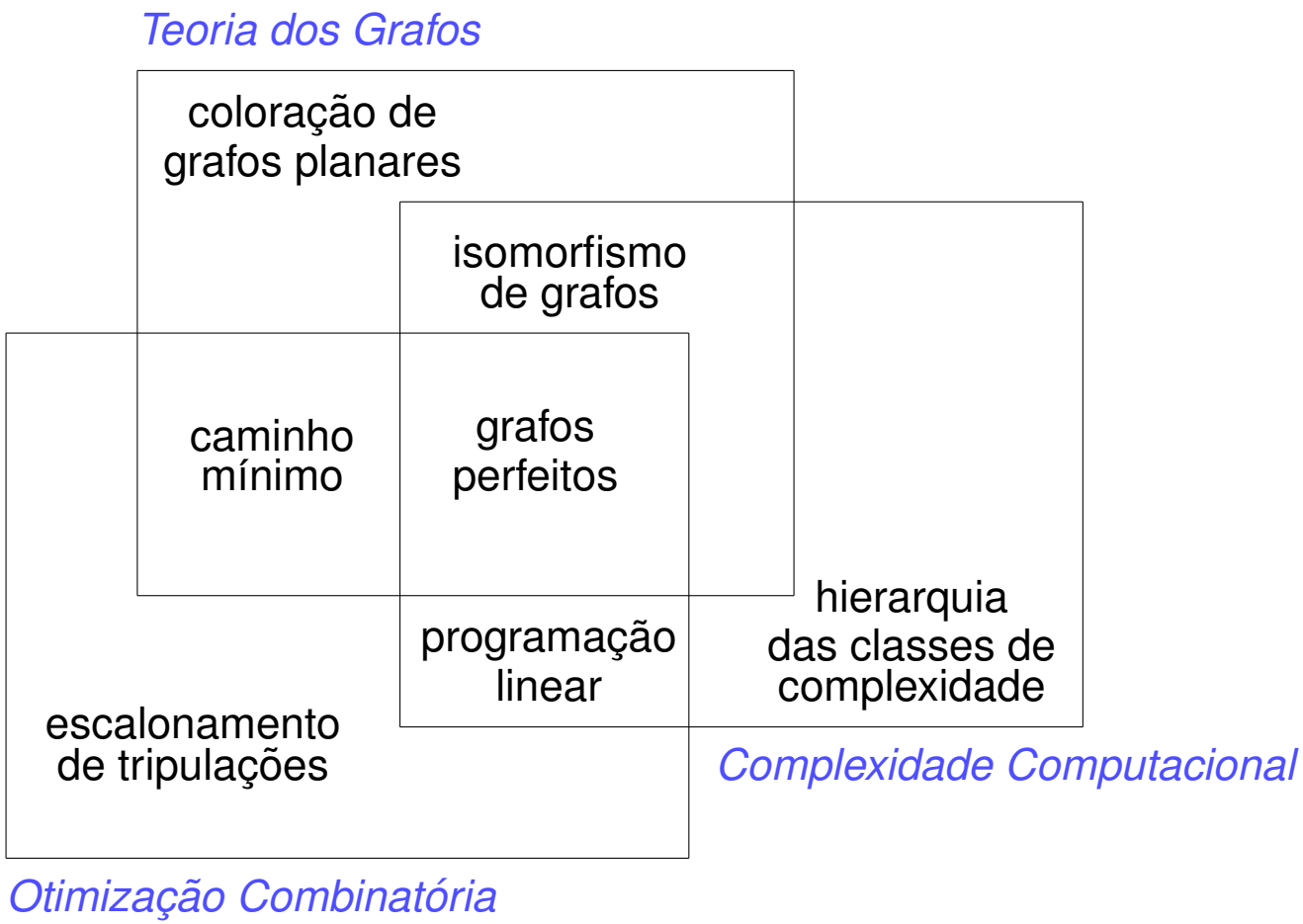


Caroline, 2021

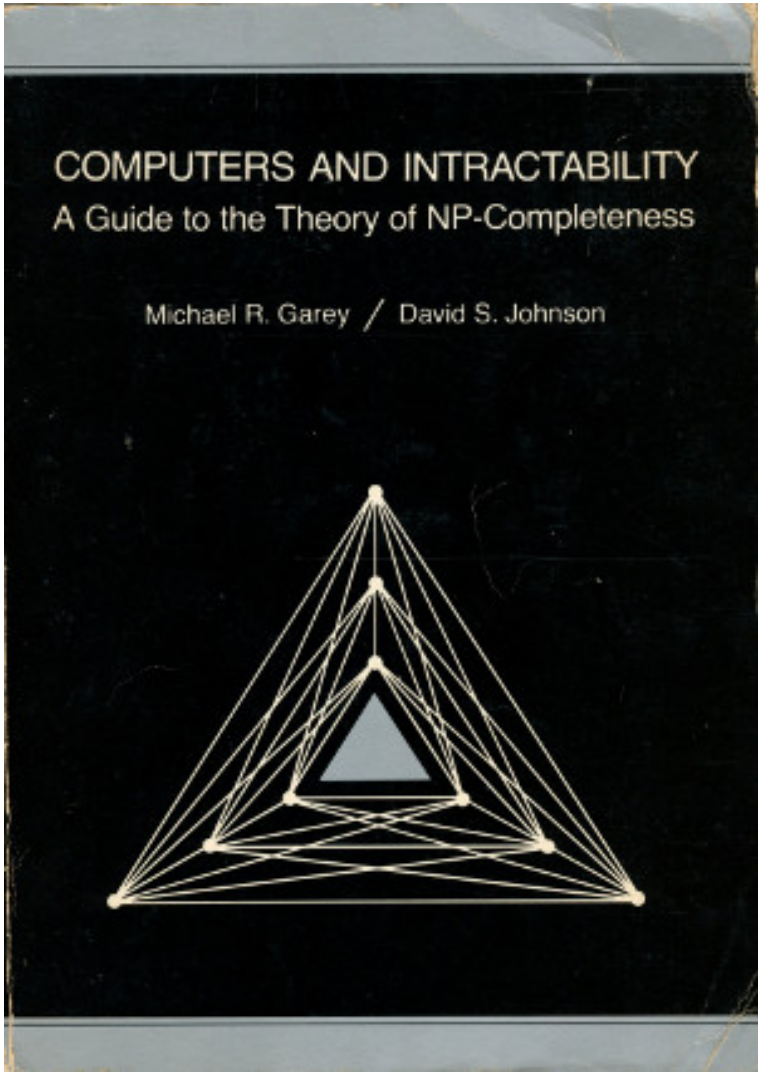


Alexsander, 2022

Origem e desenvolvimento da área de pesquisa



The Guide – Computers and Intractability



"Despite that 23 years have passed since its publication, I consider Garey and Johnson the single most important book on my office bookshelf. Every computer scientist should have this book on their shelves as well. NP-completeness is the single most important concept to come out of theoretical computer science and no book covers it as well as Garey and Johnson."

Lance Fortnow, "Great Books: Computers and Intractability: A Guide to the Theory of NP-Completeness"

Ongoing Guide – Graph Restrictions and Their Effect

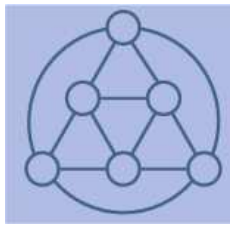
Graph Class	Member		IndSet		Clique		CliPar		ChrNum	
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]
Almost Trees (k)	P		P	[24]	P	[T]	P?		P?	
Partial k-Trees	P	[2]	P	[1]	P	[T]	P?		P	[1]
Bandwidth-k	P	[68]	P	[64]	P	[T]	P?		P	[64]
Degree-k	P	[T]	N	[GJ]	P	[T]	N	[GJ]	N	[GJ]
Planar	P	[GJ]	N	[GJ]	P	[T]	N	[10]	N	[GJ]
Series Parallel	P	[79]	P	[75]	P	[T]	P?		P	[74]
Outerplanar	P		P	[6]	P	[T]	P	[6]	P	[67]
Halin	P		P	[6]	P	[T]	P	[6]	P	[74]
k-Outerplanar	P		P	[6]	P	[T]	P	[6]	P	[6]
Grid	P		P	[GJ]	P	[T]	P	[GJ]	P	[T]
$K_{3,3}$-Free	P	[4]	N	[GJ]	P	[T]	N	[10]	N	[GJ]
Thickness-k	N	[60]	N	[GJ]	P	[T]	N	[10]	N	[GJ]
Genus-k	P	[34]	N	[GJ]	P	[T]	N	[10]	N	[GJ]
Perfect	O!		P	[42]	P	[42]	P	[42]	P	[42]
Chordal	P	[76]	P	[40]	P	[40]	P	[40]	P	[40]
Split	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]
Strongly Chordal	P	[31]	P	[40]	P	[40]	P	[40]	P	[40]
Comparability	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]
Permutation	P	[40]	P	[40]	P	[40]	P	[40]	P	[40]
Cographs	P	[T]	P	[40]	P	[40]	P	[40]	P	[40]
Undirected Path	P	[39]	P	[40]	P	[40]	P	[40]	P	[40]
Directed Path	P	[38]	P	[40]	P	[40]	P	[40]	P	[40]
Interval	P	[17]	P	[44]	P	[44]	P	[44]	P	[44]
Circular Arc	P	[78]	P	[44]	P	[50]	P	[44]	N	[36]
Circle	P	[71]	P	[GJ]	P	[50]	O?		N	[36]
Proper Circ. Arc	P	[77]	P	[44]	P	[50]	P	[44]	P	[66]
Edge (or Line)	P	[47]	P	[GJ]	P	[T]	N	[GJ]	N	[49]
Claw-Free	P	[T]	P	[63]	O?		N	[GJ]	N	[49]

ChrInd		HamCir		DomSet		MaxCut		StTree		GraIso	
P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
P?		P?		P	[45]	P?		P?		P?	
O?		P	[3]	P	[3]	P?		P?		O?	
P?		P?		P	[64]	P	[64]	P?		P	[58]
N	[49]	N	[GJ]	N	[GJ]	N	[GJ]	N	[GJ]	P	[58]
O		N	[GJ]	N	[GJ]	P	[GJ]	N	[35]	P	[GJ]
P	[74]	P	[74]	P	[54]	P	[GJ]	P	[82]	P	[GJ]
P	[67]	P	[T]	P	[6]	P	[GJ]	P	[81]	P	[GJ]
P	[74]	P	[T]	P	[6]	P	[GJ]	P?		P	[GJ]
O?		P	[6]	P	[6]	P	[GJ]	P?		P	[GJ]
P	[GJ]	N	[51]	N	[55]	P	[T]	N	[35]	P	[GJ]
O?		N	[GJ]	N	[GJ]	P	[5]	N	[GJ]	O?	
N	[49]	N	[GJ]	N	[GJ]	N	[7]	N	[GJ]	O?	
O?		N	[GJ]	N	[GJ]	O?		N	[GJ]	P	[61]
O?		N	[1]	N	[14]	O?		N	[GJ]	I	[GJ]
O?		N	[22]	N	[14]	O?		N	[83]	I	[GJ]
O?		N	[22]	N	[19]	O?		N	[83]	I	[15]
O?		O?		P	[32]	O?		P	[83]	O?	
O?		N	[1]	N	[28]	O?		N	[GJ]	I	[GJ]
P	[GJ]	N	[1]	N	[28]	P	[T]	N	[GJ]	I	[GJ]
O?		O		P	[33]	O?		P	[23]	P	[21]
O?		P	[25]	P	[33]	O?		P	[23]	P	[25]
O?		O?		N	[16]	O?		O?		I	[GJ]
O?		O?		P	[16]	O?		P	[83]	O?	
O?		P	[53]	P	[16]	O?		P	[83]	P	[57]
O?		O?		P	[13]	O?		P	[83]	O?	
O?		P	[12]	O?		O?		P	[70]	O?	
O?		P	[12]	P	[13]	O?		P	[83]	O?	
O?		N	[11]	N	[GJ]	O?		N	[70]	I	[15]
O?		N	[11]	N	[GJ]	O?		N	[70]	I	[15]

The updated NP-Completeness Column: An Ongoing Guide table 35 years later

Graph Class	Member		IndSet		Clique		CliPar		ChrNum		ChrInd		HamCir		DomSet		MaxCut		StTree		GraphIso	
Trees/Forests	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[GJ]	P	[T]	P	[GJ]
Almost Trees (k)	P	[OG]	P	[OG]	P	[T]	P	[105]	P	[5]	P	[17]	P	[5]	P	[5]	P	[20]	P	[76]	P	[17]
Partial k-trees	P	[OG]	P	[5]	P	[T]	P	[105]	P	[5]	P	[17]	P	[5]	P	[5]	P	[20]	P	[76]	P	[17]
Bandwidth-k	P	[OG]	P	[OG]	P	[T]	P	[105]	P	[5]	P	[17]	P	[5]	P	[5]	P	[OG]	P	[76]	P	[OG]
Degree-k	P	[T]	N	[GJ]	P	[T]	N	[29]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[GJ]	N	[GJ]	P	[OG]
Planar	P	[GJ]	N	[GJ]	P	[T]	N	[78]	N	[GJ]	O		N	[GJ]	N	[GJ]	P	[GJ]	N	[OG]	P	[GJ]
Series Parallel	P	[OG]	P	[OG]	P	[T]	P	[105]	P	[5]	P	[17]	P	[5]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Outerplanar	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[GJ]	P	[OG]	P	[GJ]
Halin	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[5]	P	[17]	P	[T]	P	[OG]	P	[GJ]	P	[118]	P	[GJ]
k-Outerplanar	P	[OG]	P	[OG]	P	[T]	P	[OG]	P	[5]	P	[17]	P	[OG]	P	[OG]	P	[GJ]	P	[76]	P	[GJ]
Grid	P	[OG]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[OG]	N	[32]	P	[T]	N	[OG]	P	[GJ]
$K_{3,3}$-Free*	P	[OG]	N	[GJ]	P	[T]	N	[78]	N	[GJ]	O?		N	[GJ]	N	[GJ]	P	[OG]	N	[GJ]	P	[40]
Thickness-k	N	[OG]	N	[GJ]	P	[T]	N	[78]	N	[GJ]	N	[OG]	N	[GJ]	N	[GJ]	N	[119]	N	[GJ]	I	[RJ]
Genus-k	P	[OG]	N	[GJ]	P	[T]	N	[78]	N	[GJ]	O?		N	[GJ]	N	[GJ]	O?		N	[GJ]	P	[OG]
Perfect	P	[34]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[28]	N	[OG]	N	[OG]	N	[20]	N	[GJ]	I	[84]
Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[93]	N	[OG]	N	[20]	N	[OG]	I	[84]
Split	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[93]	N	[OG]	N	[20]	N	[OG]	I	[108]
Strongly Chordal	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[93]	P	[OG]	N	[109]	P	[OG]	I	[111]
Comparability	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[28]	N	[OG]	N	[94]	N	[102]	N	[GJ]	I	[22]
Bipartite	P	[T]	P	[GJ]	P	[T]	P	[GJ]	P	[T]	P	[GJ]	N	[OG]	N	[94]	P	[T]	N	[GJ]	I	[22]
Permutation	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[44]	P	[OG]	N	[120]	P	[OG]	P	[OG]
Cographs	P	[T]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	P	[20]	P	[OG]	P	[OG]
Undirected Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[13]	N	[OG]	N	[20]	N	[RJ]	I	[22]
Directed Path	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		N	[99]	P	[OG]	N	[1]	P	[OG]	P	[7]
Interval	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	N	[1]	P	[OG]	P	[OG]
Circular Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	N	[OG]	O?		P	[106]	P	[OG]	N	[1]	P	[11]	P	[80]
Circle	P	[OG]	P	[GJ]	P	[OG]	N	[73]	N	[OG]	O?		N	[39]	N	[71]	N	[26]	P	[OG]	P	[68]
Proper Circ. Arc	P	[OG]	P	[OG]	P	[OG]	P	[OG]	P	[OG]	O?		P	[OG]	P	[OG]	O?		P	[11]	P	[82]
Edge (or Line)	P	[OG]	P	[GJ]	P	[T]	N	[95]	N	[OG]	N	[28]	N	[OG]	N	[GJ]	P	[59]	N	[19]	I	[OG]
Claw-Free	P	[T]	P	[OG]	N	[103]	N	[85]	N	[OG]	N	[28]	N	[OG]	N	[GJ]	N	[20]	N	[19]	I	[OG]

www.cos.ufrj.br/~celina/ftp/j/RJ-current.pdf



LAGOS 2009

V Latin-American Algorithms, Graphs and Optimization Symposium

The P vs. NP-complete dichotomy of some challenging problems in graph theory

Celina de Figueiredo

Universidade Federal do Rio de Janeiro

Brazil

November 2009

Two long-standing problems in graph theory

Perfect graphs: Chvátal's SKEW PARTITION is polynomial

Intersection graphs: Roberts–Spencer's CLIQUE GRAPH is NP-complete

Both SKEW PARTITION and CLIQUE GRAPH proved to be in NP when their classification into P or NP-complete was proposed

V. Chvátal – *J. Combin. Theory Ser. B* 1985

F. Roberts, J. Spencer – *J. Combin. Theory Ser. B* 1971

The three nonempty part problem

Full dichotomy for the RECOGNITION PROBLEM:

STABLE CUTSET, 3-COLORING are the only NP-complete

T. Feder, P. Hell, S. Klein, R. Motwani – *SIAM J. Discrete Math.* 2003

Full dichotomy for the SANDWICH PROBLEM:

61 interesting problems: 19 NP-complete, 42 polynomial

HOMOGENEOUS SET SANDWICH PROBLEM is polynomial

CLIQUE CUTSET SANDWICH PROBLEM is NP-complete

Full dichotomy for the GENERALIZED SPLIT GRAPH SANDWICH PROBLEM:

(2,1)-GRAPH SANDWICH PROBLEM is NP-complete

"The polynomial dichotomy for three nonempty part sandwich problems"

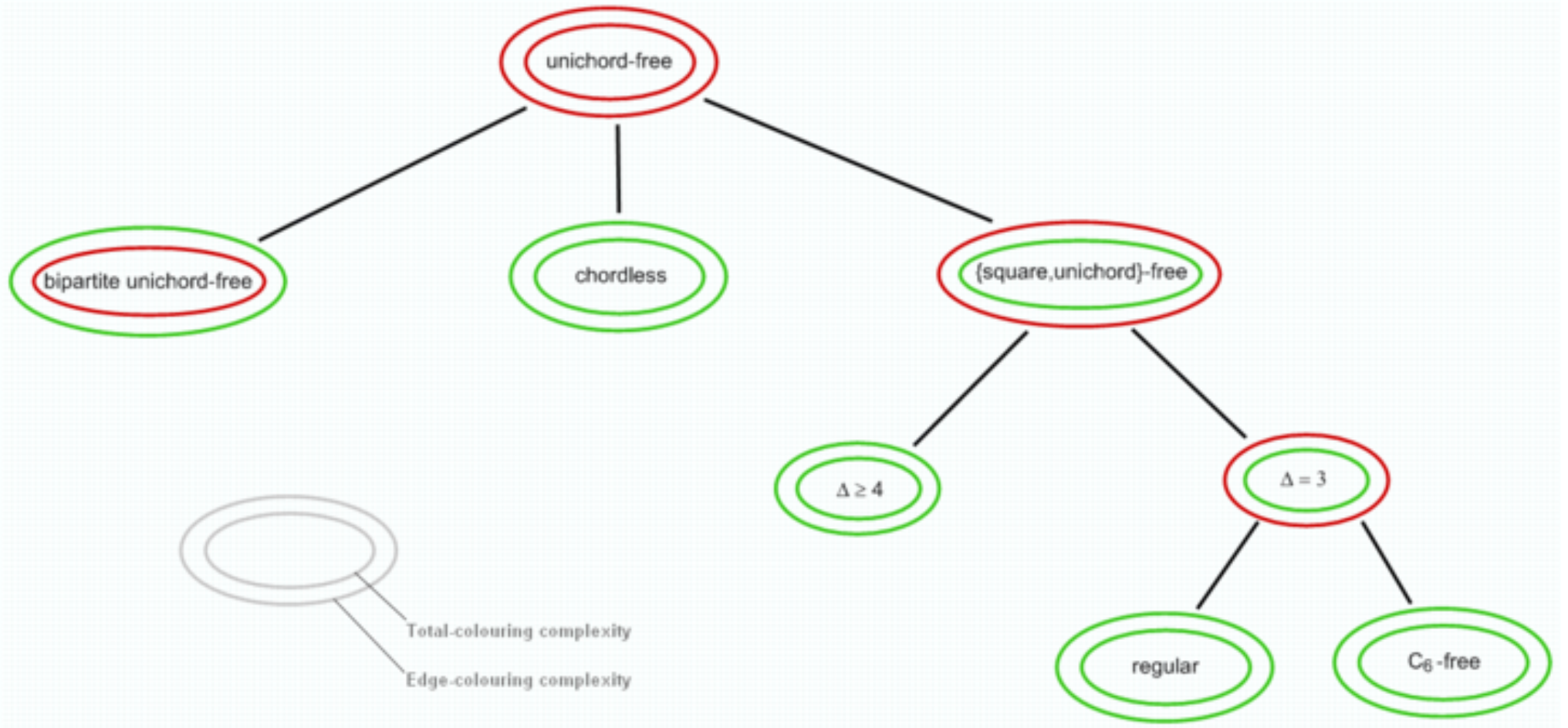
Discrete Appl. Math. 2009 (with Rafael Teixeira, Simone Dantas)

Complexity-separating graph classes for vertex, edge and total coloring

Celina de Figueiredo

COPPE
UFRJ

Edge and total coloring complexity-separating classes



When restricted to {square,unichord}-free graphs,
edge coloring is NP-complete whereas total coloring is polynomial

Complexity restricted to unichord-free and special subclasses

Colouring problem \ class	General	Unichord-free	{□, unichord}-free	{△, unichord}-free
Vertex-col.	$\mathcal{NPC}$ [14]	$\mathcal{P}$ [26]	$\mathcal{P}$ [26]	$\mathcal{P}$ [26]
Edge-col.	$\mathcal{NPC}$ [13]	$\mathcal{NPC}$ [18]	$\mathcal{NPC}$ [18]	$\mathcal{NPC}$ [18]
Total-col.	$\mathcal{NPC}$ [21]	$\mathcal{NPC}$ [17]	$\mathcal{P}$ [16,17]	$\mathcal{NPC}$ [17]
Clique-col.	$\Sigma_2^p\mathcal{C}$ [20]	$\boldsymbol{\mathcal{P}}$	$\boldsymbol{\mathcal{P}}$	$\mathcal{P}$ ($\kappa = \chi$)
Biclique-col.	$\Sigma_2^p\mathcal{C}$ [10]	$\boldsymbol{\mathcal{P}}$	$\boldsymbol{\mathcal{P}}$	$\boldsymbol{\mathcal{P}}$ ($\boldsymbol{\kappa_B = 2}$)

[10] M. Groshaus, F. Soulignac, P. Terlisky – *J. Graph Algorithms Appl.* 2014

[20] D. Marx – *Theoret. Comput. Sci.* 2011

"Efficient algorithms for clique-colouring and biclique-colouring unichord-free graphs"
Algorithmica 2017 (with Hélio Macedo and Raphael Machado)

Dániel Marx plenary talk at ICGT 2014

Every graph is easy or hard: dichotomy theorems for graph problems

Dániel Marx[1]

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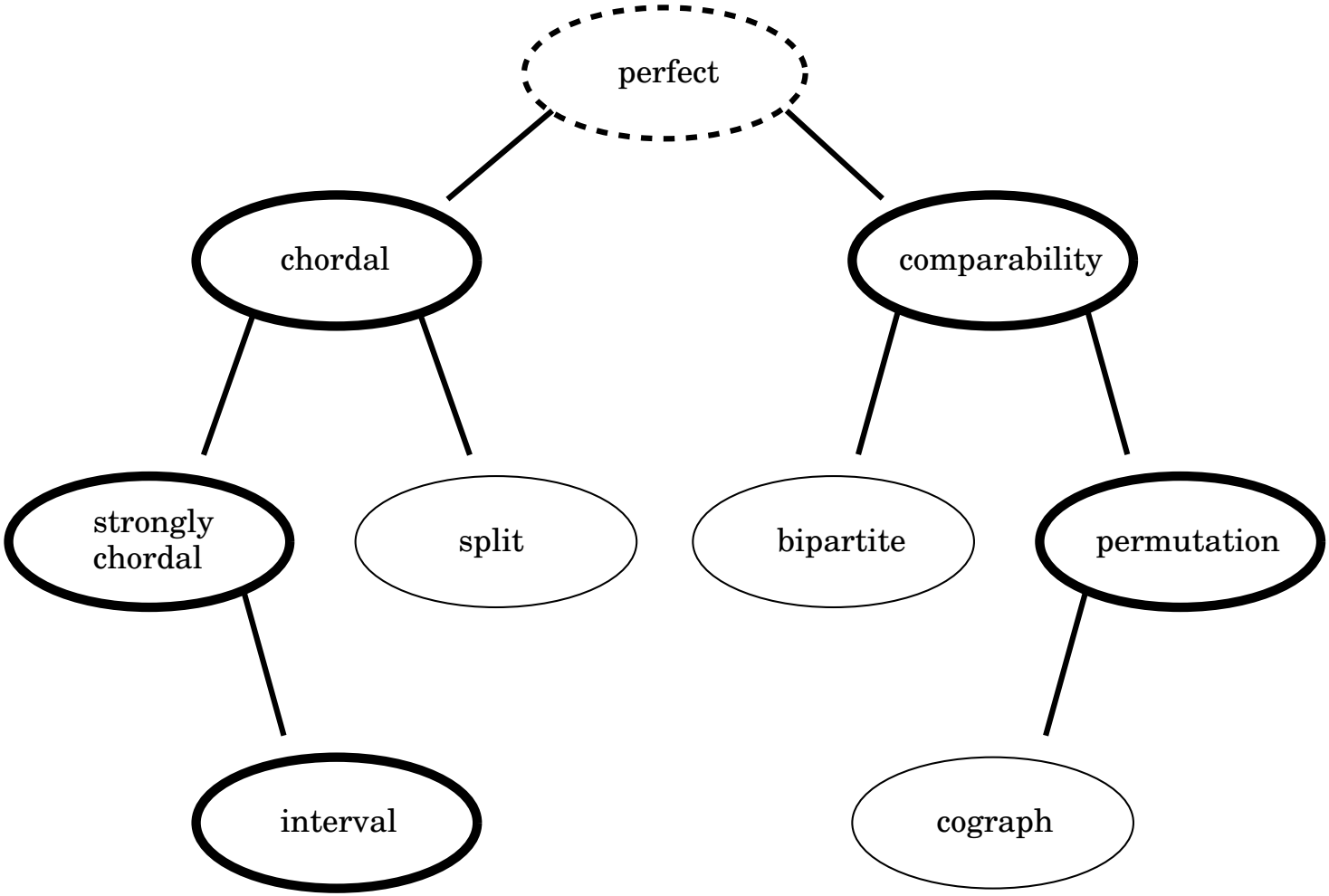
Budapest, Hungary

ICGT 2014
Grenoble, France
July 3, 2014

Sandwich problems for perfect graph classes



—— NP-complete —— polynomial - - - - - open

Is the not $\mathcal{C}$-free easier than the $\mathcal{C}$-free sandwich problem?

A trigraph (G_1, G_2) satisfies property Π if
there is no sandwich graph G for (G_1, G_2) which does not satisfy Π

The recognition of Berge graphs is polynomial

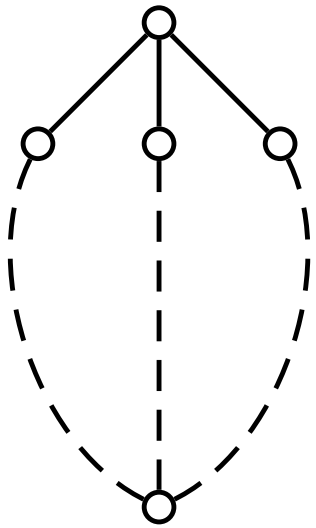
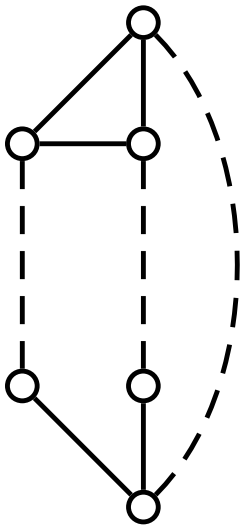
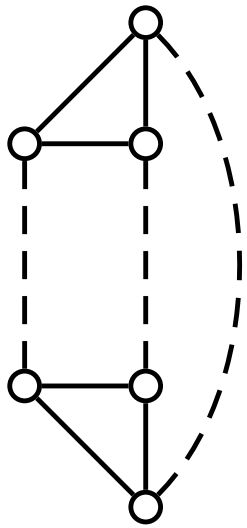
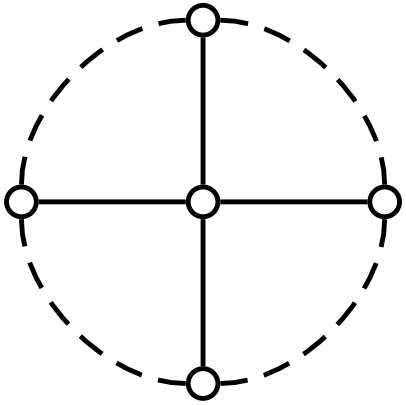
but

the recognition of Berge trigraphs was previously open

The imperfect graph sandwich problem is polynomial

Equivalently, recognizing Berge trigraphs is polynomial

Detecting 3-path configurations

theta and pyramid: polynomial

prism and wheel: NP-complete

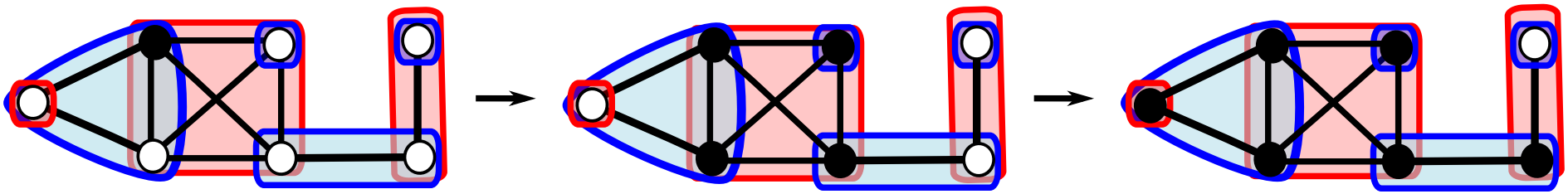
The not pyramid-free sandwich problem is polynomial

but

the complexity of the pyramid-free sandwich problem is open

"The world of hereditary graph classes viewed through Truemper configurations"
by K. Vušković, in *Surveys in Combinatorics* (2013)

A quantum walker spreads across a 2-tessellation cover



The chromatic upper bound: $T(G) \leqslant \min\{\chi'(G), \chi(K(G))\}$

"The graph tessellation cover number: Chromatic bounds, efficient algorithms and hardness" *Theoretical Computer Science* (2020) (with Alexandre Abreu, Luis Cunha, Luis Kowada, Franklin Marquezino, Daniel Posner, Renato Portugal)

Most significant publications

- FIGUEIREDO, C. M. H. · KLEIN, S. · KOHAYAKAWA, Y. · REED, B.
  Finding skew partitions efficiently
  Journal of Algorithms (2000)

- FIGUEIREDO, C. M. H. · MAFFRAY, F.
  Optimizing bull-free perfect graphs
  SIAM Journal on Discrete Mathematics (2004)

- FARIA, L. · FIGUEIREDO, C. M. H. · SYKORA, O. · VRTO, I.
  An improved upper bound on the crossing number of the hypercube
  Journal of Graph Theory (2008)

- ALCON, L. · FARIA, L. · FIGUEIREDO, C. M. H. · GUTIERREZ, M.
  The complexity of clique graph recognition
  Theoretical Computer Science (2009)

- FIGUEIREDO, C. M. H.
  The P vs. NP-complete dichotomy of some challenging problems in graph theory
  Discrete Applied Mathematics (2012)

Most significant publications

- CUNHA, L. F. I. · KOWADA, L. A. B. · HAUSEN, R. A. · FIGUEIREDO, C. M. H.
  A faster 1.375-approximation algorithm for sorting by transpositions
  Journal of Computational Biology (2015)

- MACÊDO, H. B. · MACHADO, R. C. S. · FIGUEIREDO, C. M. H.
  Hierarchical complexity of 2-clique-colouring weakly chordal graphs and perfect graphs having cliques of size at least 3
  Theoretical Computer Science (2016)

- CHUDNOVSKY, M. · FIGUEIREDO, C. M. H. · SPIRKL, S.
  The sandwich problem for decompositions and almost monotone properties
  Algorithmica (2018)

- MELO, A. A. · FIGUEIREDO, C. M. H. · SOUZA, U. S.
  A multivariate analysis of the strict terminal connection problem
  Journal of Computer and System Sciences (2020)

- ABREU, A. · CUNHA, L. · FIGUEIREDO, C. · KOWADA, L. · MARQUEZINO, F. · POSNER, D. · PORTUGAL, R.
  The graph tessellation cover number: Chromatic bounds, efficient algorithms and hardness
  Theoretical Computer Science (2020)

DOI:10.1145/3460351

Advances in algorithms, machine learning, and hardware can help tackle many NP-hard problems once thought impossible.

BY LANCE FORTNOW

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Fifty Years of P vs. NP and the Possibility of the Impossible