

# Maximum cut and Steiner tree restricted to interval graphs and related families

Celina Miraglia Herrera de Figueiredo



CANADAM 2021

## Based on

Maximum cut on interval graphs of interval count four is NP-complete

with Alexander A. de Melo, Fabiano S. Oliveira, Ana Silva

[arxiv.org/abs/2012.09804](https://arxiv.org/abs/2012.09804)



Revising Johnson's table for the 21st century

with Alexander A. de Melo, Diana Sasaki, Ana Silva

[arxiv.org/abs/2104.14395](https://arxiv.org/abs/2104.14395)



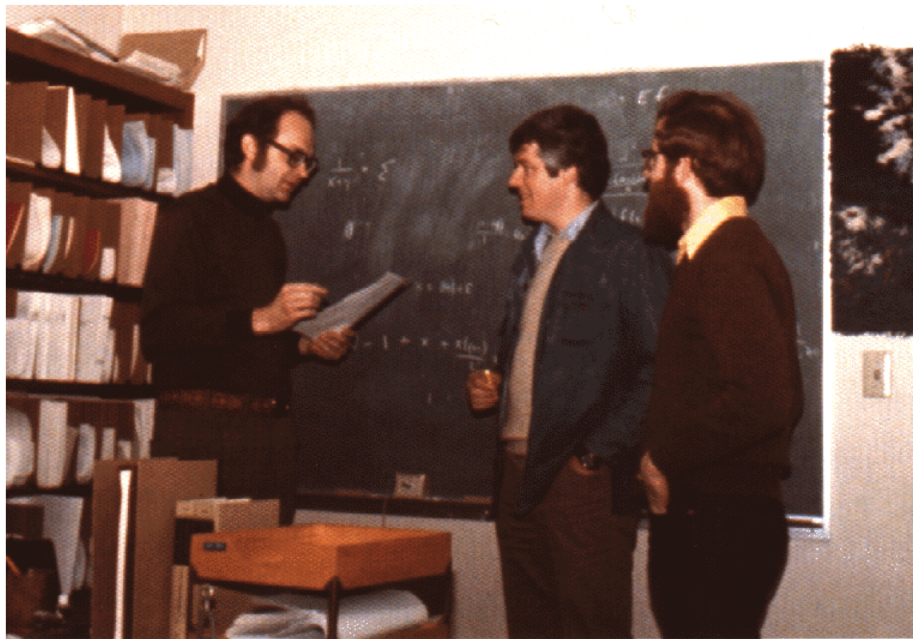
# The complexity of hard graph problems forty years later

Celina Miraglia Herrera de Figueiredo

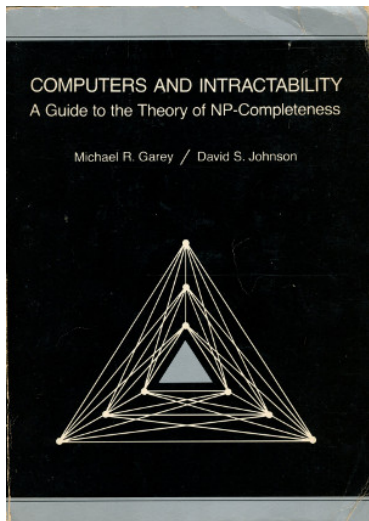


Workshop ParGO 20+50, Universidade Federal do Ceará, agosto 2019

# Knuth – Garey – Johnson



# The Guide – Computers and Intractability



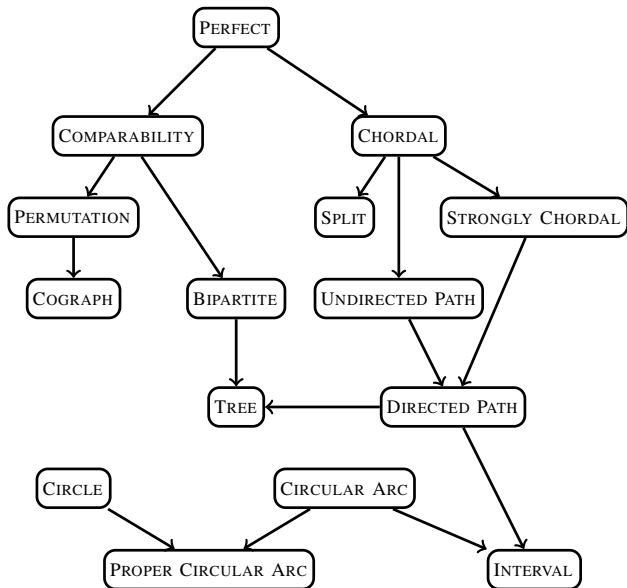
“Despite that 23 years have passed since its publication, I consider Garey and Johnson the single most important book on my office bookshelf. Every computer scientist should have this book on their shelves as well. NP-completeness is the single most important concept to come out of theoretical computer science and no book covers it as well as Garey and Johnson.”

Lance Fortnow, “Great Books: Computers and Intractability: A Guide to the Theory of NP-Completeness”

# Ongoing Guide – Graph Restrictions and Their Effect

| GRAPH CLASS          | MEMBER | INDSET | CLIQUE | CLIPAR | CHRNUM | CHRIND | HAMCIR | DOMSET | MAXCUT | STTREE | GRAISO |
|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| Trees/Forests        | P [T]  | P [GJ] | P [T]  | P [GJ] | P [T]  | P [GJ] | P [T]  | P [GJ] | P [GJ] | P [T]  | P [GJ] |
| Almost Trees ( $k$ ) | P      | P [24] | P [T]  | P?     | P?     | P?     | P?     | P [45] | P?     | P?     | P?     |
| Partial $k$ -Trees   | P [2]  | P [1]  | P [T]  | P?     | P [1]  | O?     | P [3]  | P [3]  | P?     | P?     | O?     |
| Bandwidth- $k$       | P [68] | P [64] | P [T]  | P?     | P [64] | P?     | P?     | P [64] | P [64] | P?     | P [58] |
| Degree- $k$          | P [T]  | N [GJ] | P [T]  | N [GJ] | N [GJ] | N [49] | N [GJ] | N [GJ] | N [GJ] | N [GJ] | P [58] |
| Planar               | P [GJ] | N [GJ] | P [T]  | N [10] | N [GJ] | O      | N [GJ] | N [GJ] | P [GJ] | N [35] | P [GJ] |
| Series Parallel      | P [79] | P [75] | P [T]  | P?     | P [74] | P [74] | P [74] | P [54] | P [GJ] | P [82] | P [GJ] |
| Outerplanar          | P      | P [6]  | P [T]  | P [6]  | P [67] | P [67] | P [T]  | P [6]  | P [GJ] | P [81] | P [GJ] |
| Halin                | P      | P [6]  | P [T]  | P [6]  | P [74] | P [74] | P [T]  | P [6]  | P [GJ] | P?     | P [GJ] |
| $k$ -Outerplanar     | P      | P [6]  | P [T]  | P [6]  | P [6]  | O?     | P [6]  | P [6]  | P [GJ] | P?     | P [GJ] |
| Grid                 | P      | P [GJ] | P [T]  | P [GJ] | P [T]  | P [GJ] | N [51] | N [55] | P [T]  | N [35] | P [GJ] |
| $K_{3,3}$ -Free      | P [4]  | N [GJ] | P [T]  | N [10] | N [GJ] | O?     | N [GJ] | N [GJ] | P [5]  | N [GJ] | O?     |
| Thickness- $k$       | N [60] | N [GJ] | P [T]  | N [10] | N [GJ] | N [49] | N [GJ] | N [GJ] | N [7]  | N [GJ] | O?     |
| Genus- $k$           | P [34] | N [GJ] | P [T]  | N [10] | N [GJ] | O?     | N [GJ] | N [GJ] | O?     | N [GJ] | P [61] |
| Perfect              | O!     | P [42] | P [42] | P [42] | P [42] | O?     | N [1]  | N [14] | O?     | N [GJ] | I [GJ] |
| Chordal              | P [76] | P [40] | P [40] | P [40] | P [40] | O?     | N [22] | N [14] | O?     | N [83] | I [GJ] |
| Split                | P [40] | P [40] | P [40] | P [40] | P [40] | O?     | N [22] | N [19] | O?     | N [83] | I [15] |
| Strongly Chordal     | P [31] | P [40] | P [40] | P [40] | P [40] | O?     | O?     | P [32] | O?     | P [83] | O?     |
| Comparability        | P [40] | P [40] | P [40] | P [40] | P [40] | O?     | N [1]  | N [28] | O?     | N [GJ] | I [GJ] |
| Bipartite            | P [T]  | P [GJ] | P [T]  | P [GJ] | P [T]  | P [GJ] | N [1]  | N [28] | P [T]  | N [GJ] | I [GJ] |
| Permutation          | P [40] | P [40] | P [40] | P [40] | P [40] | O?     | O      | P [33] | O?     | P [23] | P [21] |
| Cographs             | P [T]  | P [40] | P [40] | P [40] | P [40] | O?     | P [25] | P [33] | O?     | P [23] | P [25] |
| Undirected Path      | P [39] | P [40] | P [40] | P [40] | P [40] | O?     | O?     | N [16] | O?     | O?     | I [GJ] |
| Directed Path        | P [38] | P [40] | P [40] | P [40] | P [40] | O?     | O?     | P [16] | O?     | P [83] | O?     |
| Interval             | P [17] | P [44] | P [44] | P [44] | P [44] | O?     | P [53] | P [16] | O?     | P [83] | P [57] |
| Circular Arc         | P [78] | P [44] | P [50] | P [44] | N [36] | O?     | O?     | P [13] | O?     | P [83] | O?     |
| Circle               | P [71] | P [GJ] | P [50] | O?     | N [36] | O?     | P [12] | O?     | O?     | P [70] | O?     |
| Proper Circ. Arc     | P [77] | P [44] | P [50] | P [44] | P [66] | O?     | P [12] | P [13] | O?     | P [83] | O?     |
| Edge (or Line)       | P [47] | P [GJ] | P [T]  | N [GJ] | N [49] | O?     | N [11] | N [GJ] | O?     | N [70] | I [15] |
| Claw-Free            | P [T]  | P [63] | O?     | N [GJ] | N [49] | O?     | N [11] | N [GJ] | O?     | N [70] | I [15] |

# Containment relations for classes



# The updated NP-Completeness Column: An Ongoing Guide table 35 years later

| GRAPH CLASS                  | MEMBER | INDSET | CLIQUE  | CLIPAR | CHRNUM | CHRIND | HAMCIR  | DOMSET | MAXCUT  | STTREE   | GRAPHISO  |
|------------------------------|--------|--------|---------|--------|--------|--------|---------|--------|---------|----------|-----------|
| TREES/FORESTS                | P [T]  | P [GJ] | P [T]   | P [GJ] | P [T]  | P [GJ] | P [T]   | P [GJ] | P [GJ]  | P [T]    | P [GJ]    |
| ALMOST TREES ( $k$ )         | P [OG] | P [OG] | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [5]  | P [20]  | P [76]   | P [19]    |
| PARTIAL $k$ -TREES           | P [OG] | P [5]  | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [5]  | P [20]  | P [76]   | P [19]    |
| BANDWIDTH- $k$               | P [OG] | P [OG] | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [5]  | P [OG]  | P [76]   | P [OG]    |
| DEGREE- $k$                  | P [T]  | N [GJ] | P [T]   | N [29] | N [GJ] | N [OG] | N [GJ]  | N [GJ] | N [GJ]  | N [GJ]   | P [OG]    |
| PLANAR                       | P [GJ] | N [GJ] | P [T]   | N [77] | N [GJ] | O      | N [GJ]  | N [GJ] | P [GJ]  | N [OG]   | P [GJ]    |
| SERIES PARALLEL              | P [OG] | P [OG] | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [OG] | P [GJ]  | P [OG]   | P [GJ]    |
| OUTERPLANAR                  | P [OG] | P [OG] | P [T]   | P [OG] | P [OG] | P [OG] | P [T]   | P [OG] | P [GJ]  | P [OG]   | P [GJ]    |
| HALIN                        | P [OG] | P [OG] | P [T]   | P [OG] | P [5]  | P [19] | P [T]   | P [OG] | P [GJ]  | P [118]  | P [GJ]    |
| $k$ -OUTERPLANAR             | P [OG] | P [OG] | P [T]   | P [OG] | P [5]  | P [19] | P [OG]  | P [OG] | P [GJ]  | P [76]   | P [GJ]    |
| GRID                         | P [OG] | P [GJ] | P [T]   | P [GJ] | P [T]  | P [GJ] | N [OG]  | N [32] | P [T]   | N [OG]   | P [GJ]    |
| $K_{3,3}$ -FREE <sup>+</sup> | P [OG] | N [GJ] | P [T]   | N [77] | N [GJ] | O?     | N [GJ]  | N [GJ] | P [OG]  | N [GJ]   | P [40]    |
| THICKNESS- $k$               | N [OG] | N [GJ] | P [T]   | N [77] | N [GJ] | N [OG] | N [GJ]  | N [GJ] | N [119] | N [GJ]   | I Prop. 3 |
| GENUS- $k$                   | P [OG] | N [GJ] | P [T]   | N [77] | N [GJ] | O?     | N [GJ]  | N [GJ] | O?      | N [GJ]   | P [OG]    |
| PERFECT                      | P [34] | P [OG] | P [OG]  | P [OG] | P [OG] | N [28] | N [OG]  | N [OG] | N [20]  | N [GJ]   | I [83]    |
| CHORDAL                      | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [92]  | N [OG] | N [20]  | N [OG]   | I [83]    |
| SPLIT                        | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [92]  | N [OG] | N [20]  | N [OG]   | I [107]   |
| STRONGLY CHORDAL             | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [92]  | P [OG] | N [108] | P [OG]   | I [111]   |
| COMPARABILITY                | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | N [28] | N [OG]  | N [93] | N [101] | N [GJ]   | I [23]    |
| BIPARTITE                    | P [T]  | P [GJ] | P [T]   | P [GJ] | P [T]  | P [GJ] | N [OG]  | N [93] | P [T]   | N [GJ]   | I [23]    |
| PERMUTATION                  | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [44]  | P [OG] | O?      | P [OG]   | P [OG]    |
| COGRAPHS                     | P [T]  | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [OG]  | P [OG] | P [20]  | P [OG]   | P [OG]    |
| UNDIRECTED Path              | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [13]  | N [OG] | N [20]  | N Thm. 4 | I [23]    |
| DIRECTED PATH                | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [98]  | P [OG] | N [1]   | P [OG]   | P [7]     |
| INTERVAL                     | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [OG]  | P [OG] | N [1]   | P [OG]   | P [OG]    |
| CIRCULAR ARC                 | P [OG] | P [OG] | P [OG]  | P [OG] | N [OG] | O?     | P [105] | P [OG] | N [1]   | P [11]   | P [79]    |
| CIRCLE                       | P [OG] | P [GJ] | P [OG]  | N [73] | N [OG] | O?     | N [39]  | N [71] | N [26]  | P [OG]   | P [68]    |
| PROPER CIRC. ARC             | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [OG]  | P [OG] | O?      | P [11]   | P [81]    |
| EDGE (OR LINE)               | P [OG] | P [GJ] | P [T]   | N [94] | N [OG] | N [28] | N [OG]  | N [GJ] | P [59]  | N [18]   | I [OG]    |
| CLAW-FREE                    | P [T]  | P [OG] | N [102] | N [84] | N [OG] | N [28] | N [OG]  | N [GJ] | N [20]  | N [18]   | I [OG]    |



# Maximum cut and Steiner tree

We focus on columns MaxCut and StTree

Full dichotomy for StTree column: the problem is NP-complete when restricted to UNDIRECTED PATH graphs

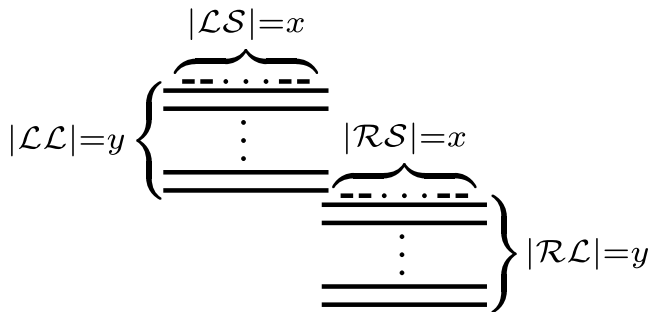
## Complexity of maximum cut on interval graphs

R. Adhikary, K. Bose, S. Mukherjee, and B. Roy.

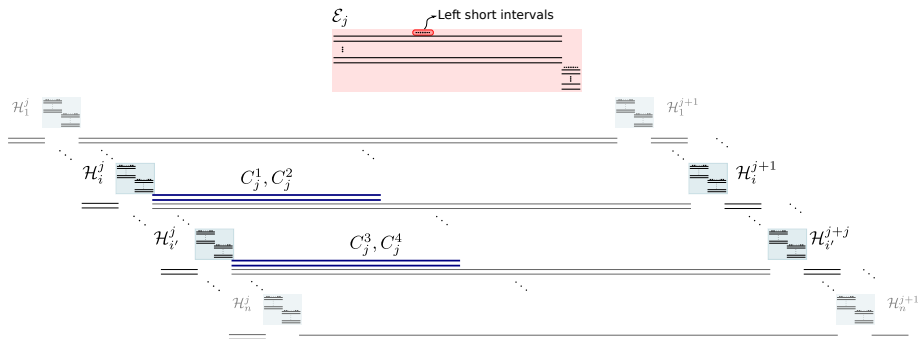
[arxiv.org/abs/2006.00061](https://arxiv.org/abs/2006.00061)

MaxCut restricted to interval graphs with bounded interval count is NP-complete

# The grained gadget of Adhikary et al.



# Our escalator gadget



# The updated NP-Completeness Column: An Ongoing Guide table 35 years later

| GRAPH CLASS                  | MEMBER | INDSET | CLIQUE  | CLIPAR | CHRNUM | CHRIND | HAMCIR  | DOMSET | MAXCUT  | STTREE   | GRAPHISO  |
|------------------------------|--------|--------|---------|--------|--------|--------|---------|--------|---------|----------|-----------|
| TREES/FORESTS                | P [T]  | P [GJ] | P [T]   | P [GJ] | P [T]  | P [GJ] | P [T]   | P [GJ] | P [GJ]  | P [T]    | P [GJ]    |
| ALMOST TREES ( $k$ )         | P [OG] | P [OG] | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [5]  | P [20]  | P [76]   | P [19]    |
| PARTIAL $k$ -TREES           | P [OG] | P [5]  | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [5]  | P [20]  | P [76]   | P [19]    |
| BANDWIDTH- $k$               | P [OG] | P [OG] | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [5]  | P [OG]  | P [76]   | P [OG]    |
| DEGREE- $k$                  | P [T]  | N [GJ] | P [T]   | N [29] | N [GJ] | N [OG] | N [GJ]  | N [GJ] | N [GJ]  | N [GJ]   | P [OG]    |
| PLANAR                       | P [GJ] | N [GJ] | P [T]   | N [77] | N [GJ] | O      | N [GJ]  | N [GJ] | P [GJ]  | N [OG]   | P [GJ]    |
| SERIES PARALLEL              | P [OG] | P [OG] | P [T]   | P [16] | P [5]  | P [19] | P [5]   | P [OG] | P [GJ]  | P [OG]   | P [GJ]    |
| OUTERPLANAR                  | P [OG] | P [OG] | P [T]   | P [OG] | P [OG] | P [OG] | P [T]   | P [OG] | P [GJ]  | P [OG]   | P [GJ]    |
| HALIN                        | P [OG] | P [OG] | P [T]   | P [OG] | P [5]  | P [19] | P [T]   | P [OG] | P [GJ]  | P [118]  | P [GJ]    |
| $k$ -OUTERPLANAR             | P [OG] | P [OG] | P [T]   | P [OG] | P [5]  | P [19] | P [OG]  | P [OG] | P [GJ]  | P [76]   | P [GJ]    |
| GRID                         | P [OG] | P [GJ] | P [T]   | P [GJ] | P [T]  | P [GJ] | N [OG]  | N [32] | P [T]   | N [OG]   | P [GJ]    |
| $K_{3,3}$ -FREE <sup>+</sup> | P [OG] | N [GJ] | P [T]   | N [77] | N [GJ] | O?     | N [GJ]  | N [GJ] | P [OG]  | N [GJ]   | P [40]    |
| THICKNESS- $k$               | N [OG] | N [GJ] | P [T]   | N [77] | N [GJ] | N [OG] | N [GJ]  | N [GJ] | N [119] | N [GJ]   | I Prop. 3 |
| GENUS- $k$                   | P [OG] | N [GJ] | P [T]   | N [77] | N [GJ] | O?     | N [GJ]  | N [GJ] | O?      | N [GJ]   | P [OG]    |
| PERFECT                      | P [34] | P [OG] | P [OG]  | P [OG] | P [OG] | N [28] | N [OG]  | N [OG] | N [20]  | N [GJ]   | I [83]    |
| CHORDAL                      | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [92]  | N [OG] | N [20]  | N [OG]   | I [83]    |
| SPLIT                        | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [92]  | N [OG] | N [20]  | N [OG]   | I [107]   |
| STRONGLY CHORDAL             | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [92]  | P [OG] | N [108] | P [OG]   | I [111]   |
| COMPARABILITY                | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | N [28] | N [OG]  | N [93] | N [101] | N [GJ]   | I [23]    |
| BIPARTITE                    | P [T]  | P [GJ] | P [T]   | P [GJ] | P [T]  | P [GJ] | N [OG]  | N [93] | P [T]   | N [GJ]   | I [23]    |
| PERMUTATION                  | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [44]  | P [OG] | O?      | P [OG]   | P [OG]    |
| COGRAPHS                     | P [T]  | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [OG]  | P [OG] | P [20]  | P [OG]   | P [OG]    |
| UNDIRECTED Path              | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [13]  | N [OG] | N [20]  | N Thm. 4 | I [23]    |
| DIRECTED PATH                | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | N [98]  | P [OG] | N [1]   | P [OG]   | P [7]     |
| INTERVAL                     | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [OG]  | P [OG] | N [1]   | P [OG]   | P [OG]    |
| CIRCULAR ARC                 | P [OG] | P [OG] | P [OG]  | P [OG] | N [OG] | O?     | P [105] | P [OG] | N [1]   | P [11]   | P [79]    |
| CIRCLE                       | P [OG] | P [GJ] | P [OG]  | N [73] | N [OG] | O?     | N [39]  | N [71] | N [26]  | P [OG]   | P [68]    |
| PROPER CIRC. ARC             | P [OG] | P [OG] | P [OG]  | P [OG] | P [OG] | O?     | P [OG]  | P [OG] | O?      | P [11]   | P [81]    |
| EDGE (OR LINE)               | P [OG] | P [GJ] | P [T]   | N [94] | N [OG] | N [28] | N [OG]  | N [GJ] | P [59]  | N [18]   | I [OG]    |
| CLAW-FREE                    | P [T]  | P [OG] | N [102] | N [84] | N [OG] | N [28] | N [OG]  | N [GJ] | N [20]  | N [18]   | I [OG]    |

# Why is ChrInd the hardest of Johnson's puzzles?

## The chromatic index of split-interval graphs

Luis Gustavo Gonzaga, Sheila Almeida, Cândida Silva and Jadder Cruz

presented last week at LAGOS 2021 – XI Latin and American Algorithms,  
Graphs and Optimization Symposium

[eventos.ufabc.edu.br/lagos2021/](https://eventos.ufabc.edu.br/lagos2021/)



# Greetings from Latin America

