ALGORITHMS FOR QUANTUM COMPUTERS

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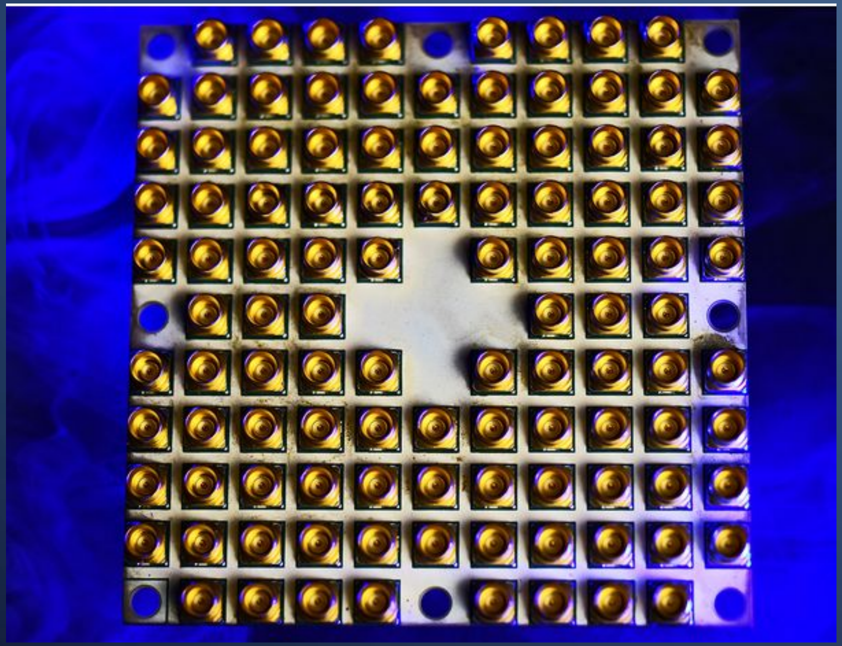
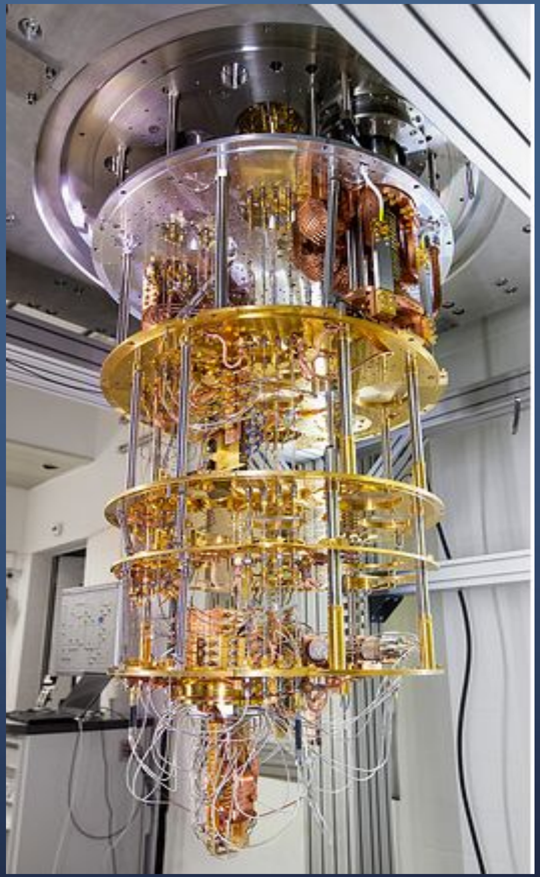
SMALL SCALE QUANTUM COMPUTERS



Intel, 49 qubits



IBM, 50 qubits

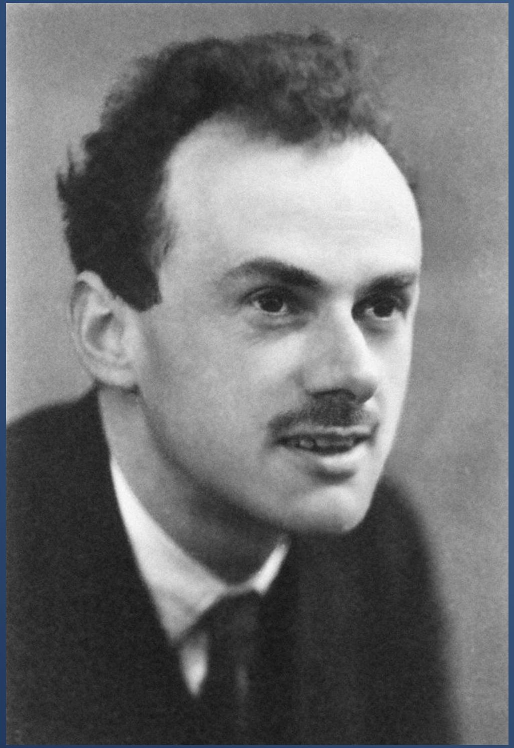
Google, 72 qubits

OTHER QUANTUM COMPUTING EFFORTS

QUANTUM SIMULATION

NUMBER OF VARIABLES

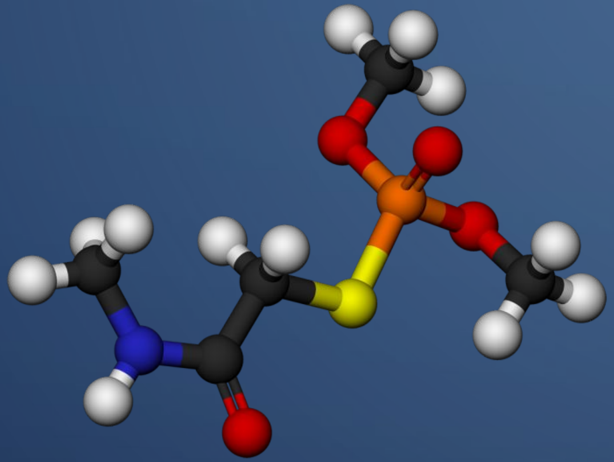
- 30 particles – $2^{30} \approx 1{,}000{,}000{,}000$.
- 100 particles – $2^{100} \approx \underbrace{1{,}000 \ldots 000}_{\text{30 zeros}}$.



Paul Dirac, 1929

The underlying physical laws ... for a large part of physics and the whole chemistry are thus completely known and the difficulty is only that the exact application of these laws leads to equations much too complicated to be soluble.

QUANTUM CHEMISTRY

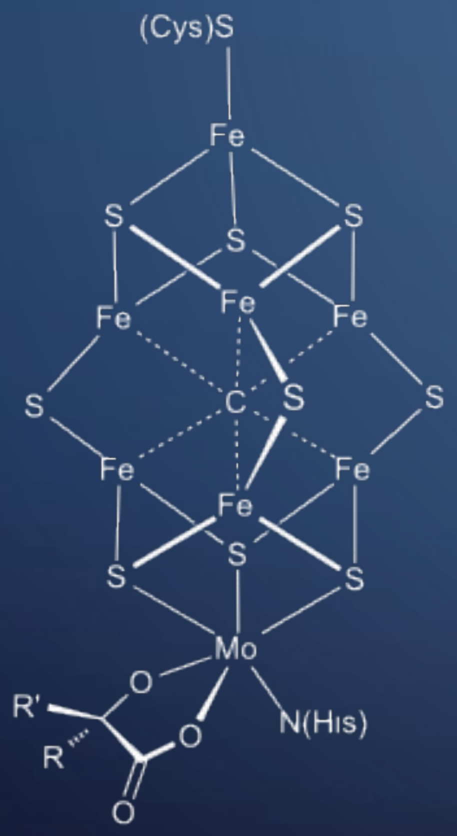


$$H|\Psi\rangle = E|\Psi\rangle$$

$$H = \sum_{p,q} t_{pq} c_p^+ c_q + \sum_{p,q,r,s} V_{pqrs} c_p^+ c_q^+ c_r c_s$$

A substantial application for HPC

EXAMPLE: NITROGEN FIXATION



- $N_2 \rightarrow 2\ NH_3$.
- Performed at room temperature by nitrogenase enzyme in plants.
- The active site (FeMoCo) can be modelled with 110-2000 qubits.

Reiher et al., Elucidating reaction mechanisms on quantum computers, PNAS, 2017:201619152.

FACTORING AND CODE-BREAKING

FACTORING

6231540623 = 93599 * 66577.

Given 6231540623, find factors?

For large (300 digit) numbers conventional computers are too slow.

Shor, 1994: quantum computers can factor large numbers efficiently.

IMPLICATIONS

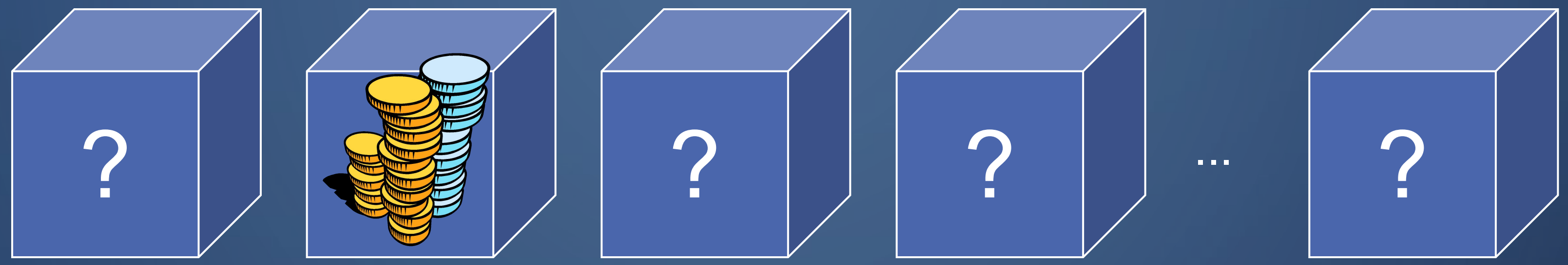
- RSA and other cryptosystems based on factoring/discrete log broken by quantum computers.
- Quantum algorithms for other crypto-related problems:
  - Pell's equation [Hallgren, 2002],
  - ideal class group [Biasse, Song, 2016],
  - principal ideal problem [Hallgren, 2002, Biasse, Song, 2016].

POST QUANTUM CRYPTOGRAPHY

- Classical schemes secure against quantum attacks:
  - Lattice based (e.g. NTRU, LWE);
  - Code based (e.g. McEliece);
  - Based on multivariate quadratic equations (oil-and-vinegar);
- NIST Call for Proposals for Post Quantum Crypto Standard: Nov 30, 2017.

SEARCH

QUANTUM SEARCH



- N objects;
- Find an object with a certain property.

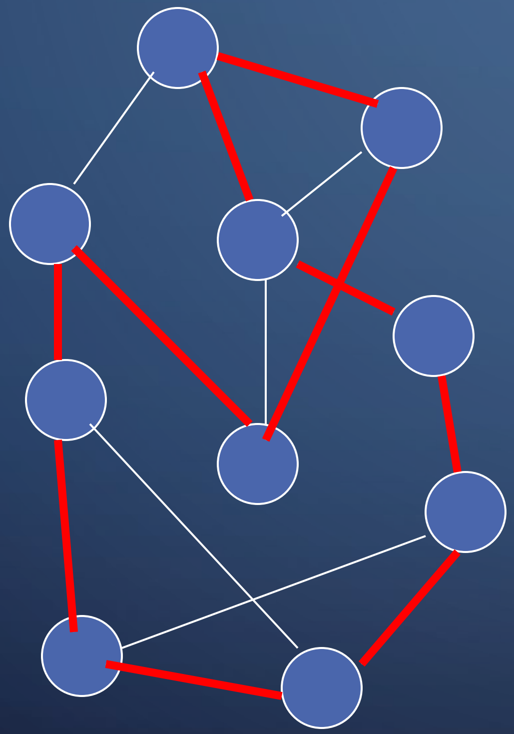
Grover, 1996: O(√N) quantum steps.

Who has the number 67033706?

Usual computer: N = 1,000,000 steps

Quantum computer: $\sqrt{N}$ = 1000

APPLICATION: TRAVELING SALESMAN PROBLEM



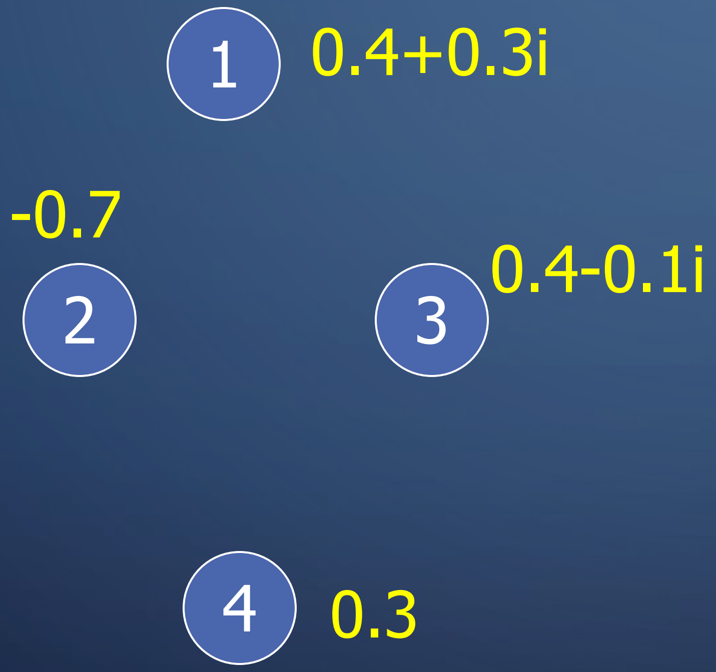
Find the best route through all the cities.

Quantum algorithm:

- 1,000,000 candidates;
- Time = 1,000

THE SOURCE OF QUANTUM ADVANTAGE?

QUANTUM SYSTEM



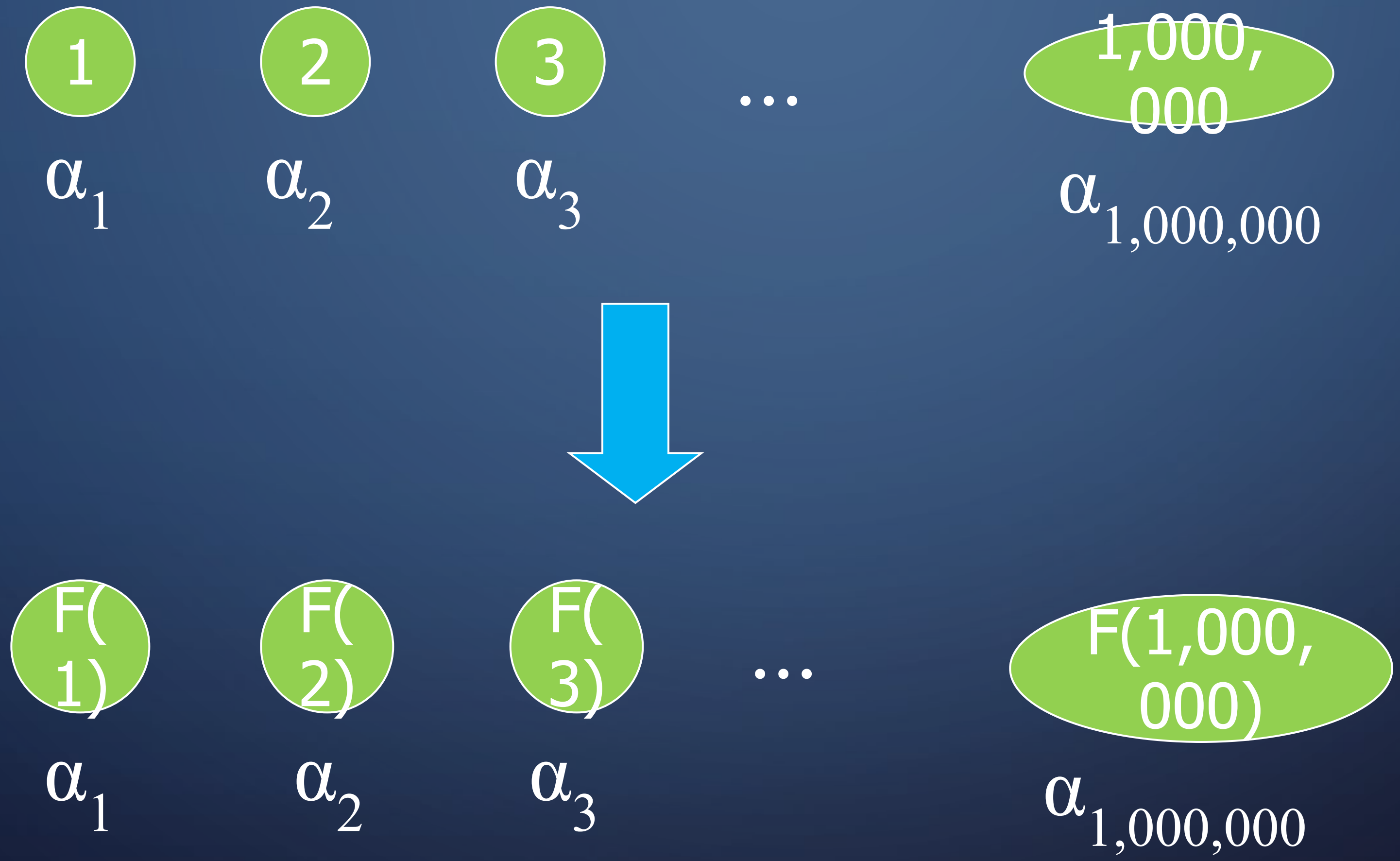
Current state: i with amplitude α_i.

$$\sum_i |\alpha_i|^2 = 1$$

What is it [quantum mechanics] about? From my perspective, it's about information and probabilities and observables, and how they relate to each other.

/Scott Aaronson, MIT, 2005/

QUANTUM PARALLELISM



WHAT CAN WE DO WITH SUCH STATE?

F(1) F(2) F(3) … F(1,000,000)

α_1 α_2 α_3 $\alpha_{1,000,000}$

Measurement

F(3)

Other values are lost!

QUANTUM INTERFERENCE

F(1) F(2) F(3) … F(1,000,000)

α_1 α_2 α_3 $\alpha_{1,000,000}$

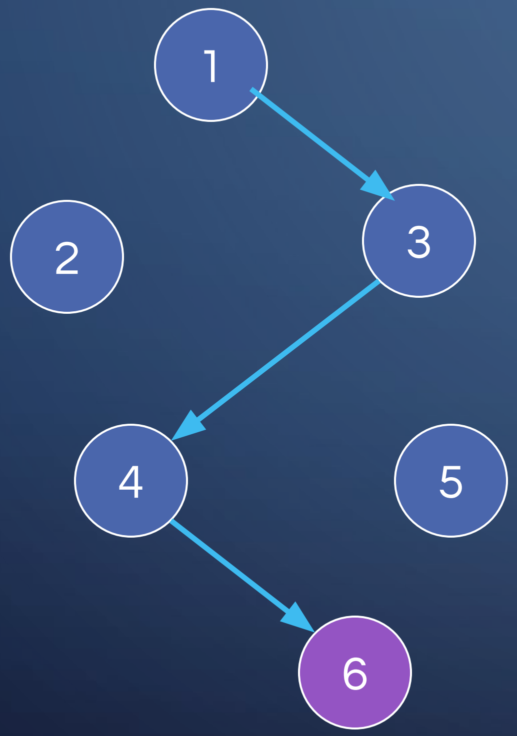
Result that depends on all values
F(1), F(2), …

Example: quantum Fourier transform finds periods quickly

QUANTUM COMPUTERS WILL PROVIDE BIG SPEEDUPS FOR SOME BUT NOT ALL COMPUTATIONAL TASKS

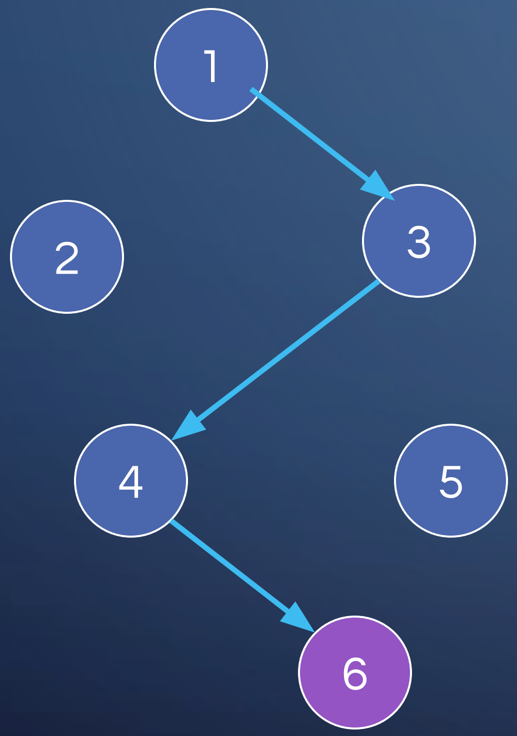
QUANTUM SPEEDUP FOR SMART SEARCH?

QUANTUM WALK SEARCH [SZEGEDY, 04]



- Finite search space.
- Some elements might be marked.
- Find a marked element!
  - Perform a random walk, stop after finding a marked element.

CONDITIONS ON MARKOV CHAIN



- Random walk must be symmetric: $p_{xy}=p_{yx}$.
- Start state = uniformly random state.
- T = expected time to reach marked state, if there is one.

MAIN RESULT [SZEGEDY, 04]

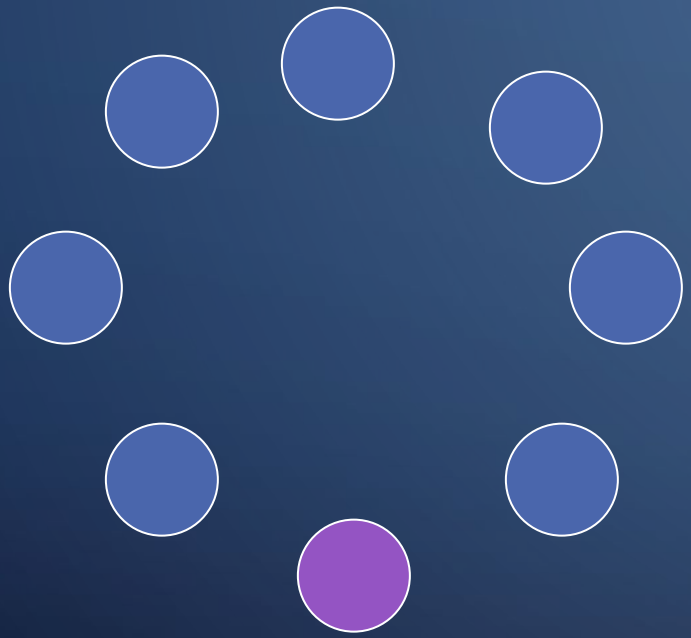
Theorem Assume that:

1. There are no marked states, or
2. A marked state is reached in expected time at most T.

A quantum algorithm can distinguish the two cases in time $O(\sqrt{T})$.

Quadratic speedup for a variety of problems.

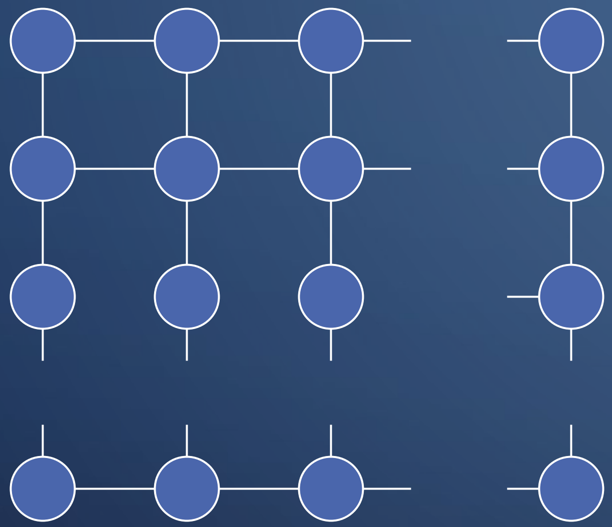
APPLICATION 1



- N states.
- Is there a marked state?
- Random walk: at each step move to a randomly chosen vertex.
- Finds a marked vertex in N expected steps.

Quantum: O(√N) steps [Grover]

APPLICATION 2: SEARCH ON GRIDS



- Random walk: at each step move to a random neighbour.
- Finding marked state: O(N log N) steps.

Quantum algorithm: [A, Kempe, Rivosh, 2005]

$$O\left(\sqrt{N \log N}\right)$$

APPLICATION 3: ELEMENT DISTINCTNESS

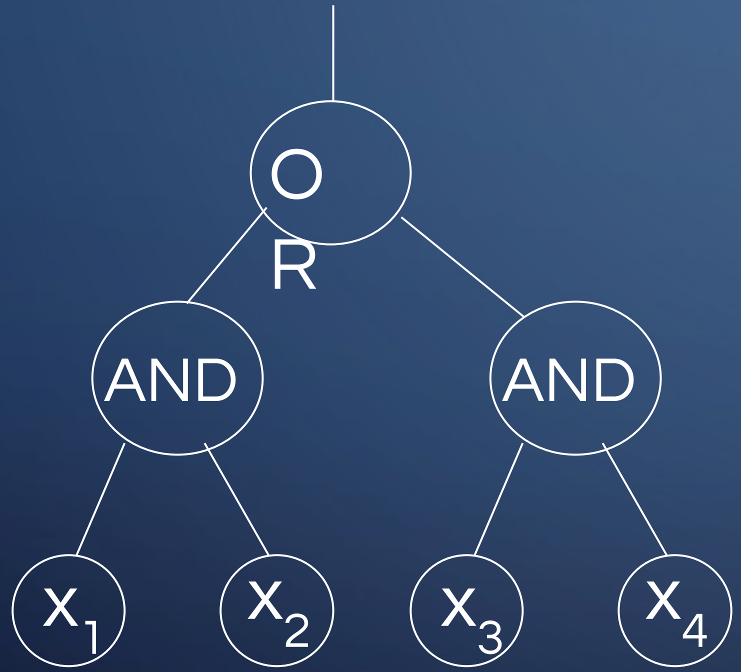
31 40 75 71 93 32 47
11 70 37 78 79 36 63
40 48 98 23 41 16 66
75 38 27 42 55 77 19
45 15 53 22 91 37 90
58 13 10 25 29 25 56
68 12 11 51 23 77 15
17 24 88 56 42

- Are there 2 equal numbers?
- Usual algorithms: N steps.
- Quantum algorithms: $O(N^{2/3})$.

[A, 2004]

QUANTUM ALGORITHMS FOR LOGIC FORMULAS

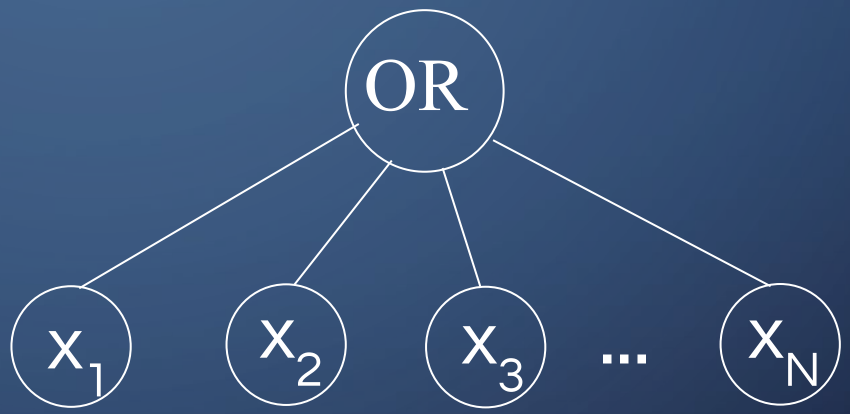
EVALUATING AND-OR TREES



- Variables x_i accessed by queries to a black box:
  - Input i;
  - Black box outputs x_i.
- Evaluate T with the smallest number of queries.

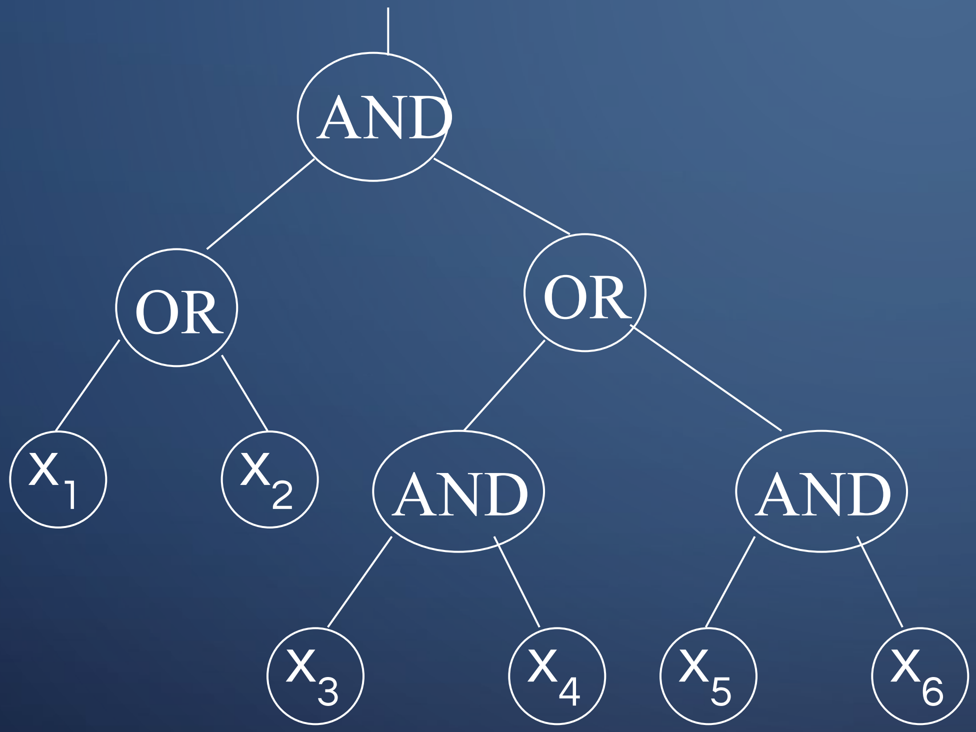
SEARCH AS FORMULA EVALUATION

- Is there i: $x_i=1$?



Quantum algorithm with $O(\sqrt{N})$ queries.

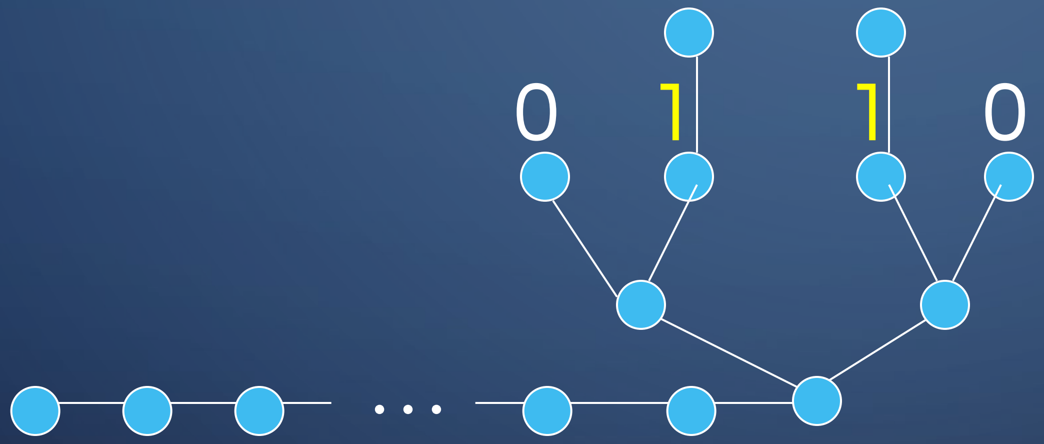
LOGIC FORMULAS



- Formula size: N.
- Quantum: $\Theta(\sqrt{N})$ queries.

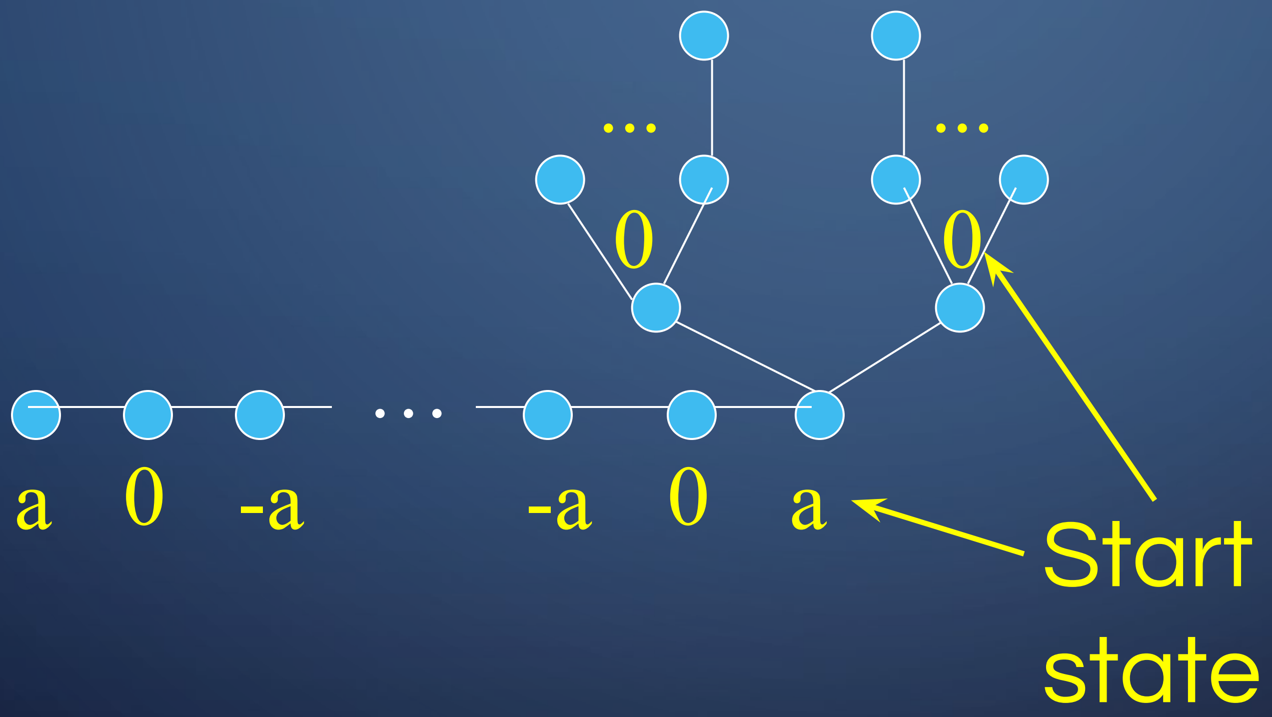
*[Farhi, Gutman, Goldstone, 2007,
A, Childs, Špalek, Reichardt, Zhang, 2007, Reichardt, 2010]*

FORMULA TREE

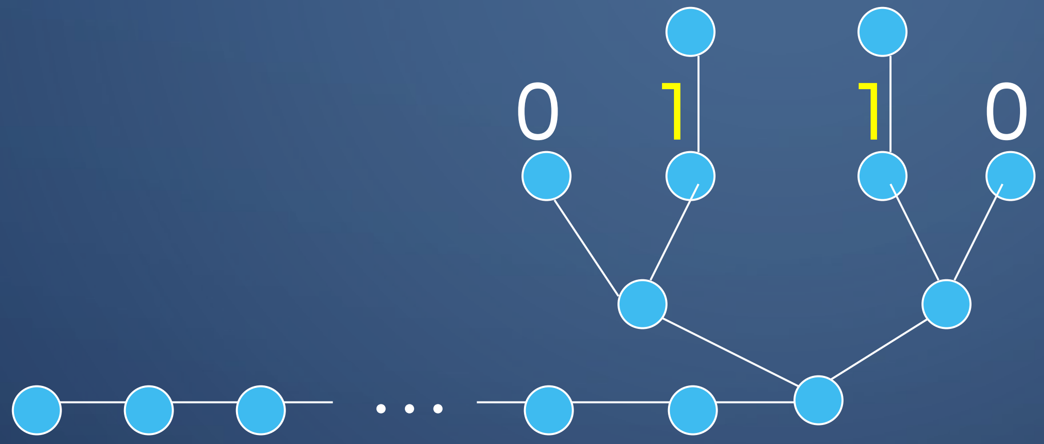


Finite
“tail”

ALGORITHM



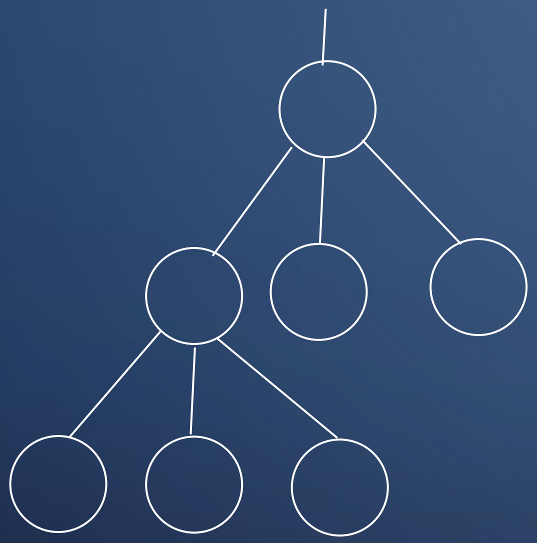
WHAT HAPPENS?



- If F=0, state is (almost) unchanged.
- If F=1, state «scatters» into the tree.

QUANTUM SEARCH ON TREES

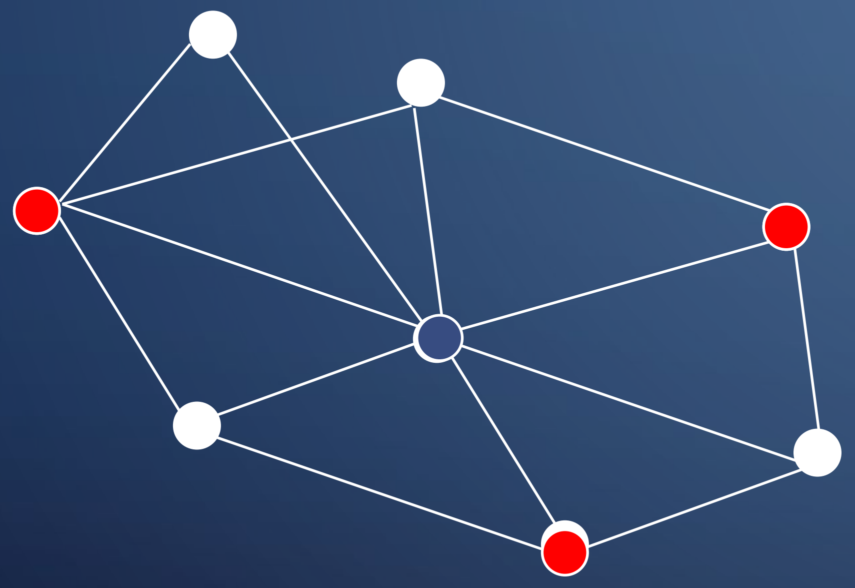
BACKTRACKING



- Start with a partial solutions, try to expand it.
- Applications: SAT solvers, optimization, etc.

More difficult than simple exhaustive search.

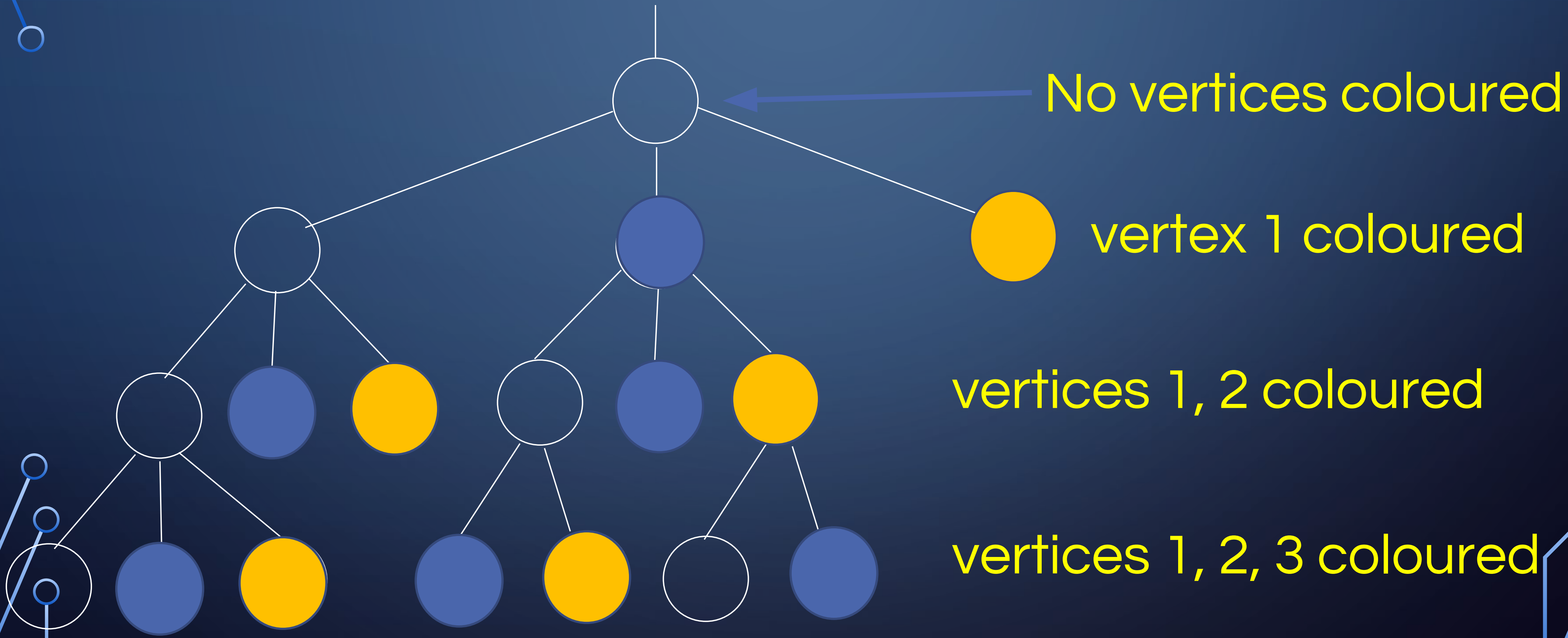
EXAMPLE



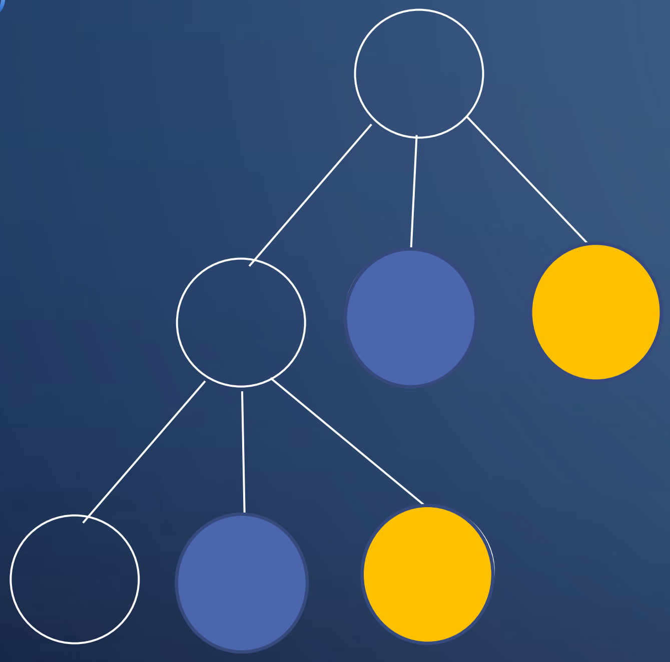
- 3-COLORING: Can we colour vertices with 3 colours so that no edge is monochromatic?
- NP-complete.

Algorithm: attempt to colour vertices one by one.

TREE OF PARTIAL COLORINGS



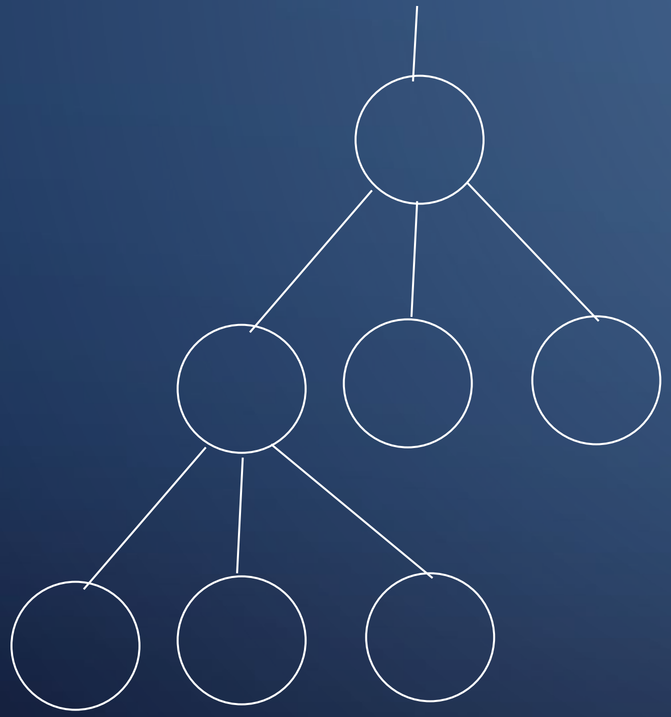
STANDARD QUANTUM SEARCH?



- Grover requires:
  - being able to enumerate search space;
  - being able to find i^{th} candidate solution.

- Because of irregular structure:
  - Some vertices have more children;

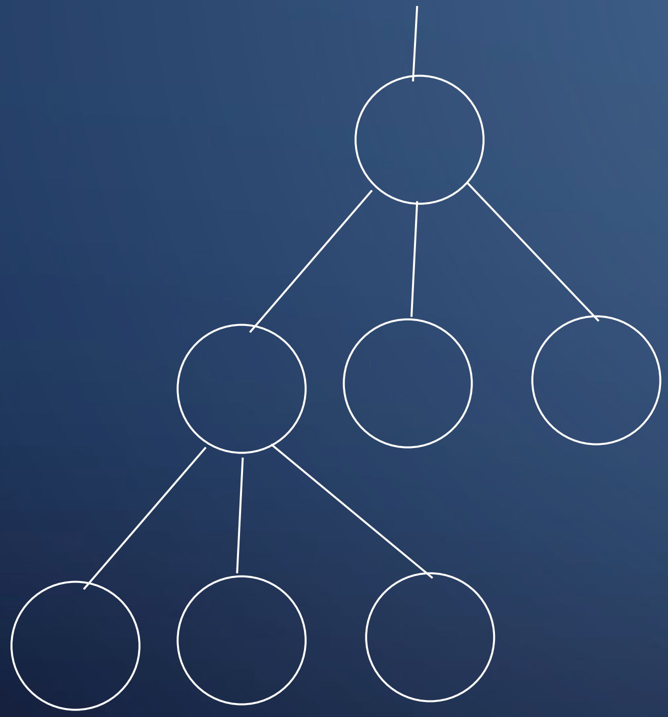
QUANTUM ALGORITHM



- Quantum algorithm for searching a tree in time $O(\sqrt{Tn})$,
  - T – size of the tree;
  - n – depth of the tree.
- Almost quadratic advantage.

[Montanaro, 2015, A, Kokainis, 2017]

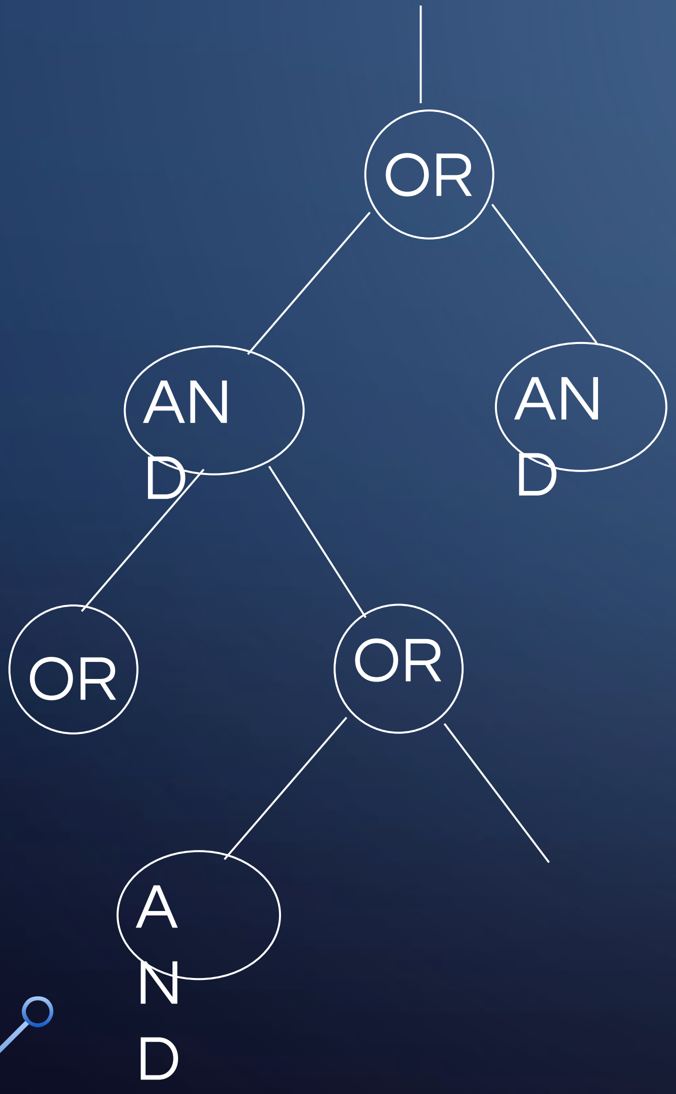
SHORT TERM EXPERIMENT?



- Small memory requirements:
  - Basis states = vertices;
  - N vertex tree = log N qubits.

SEARCHING GAME TREES

POSITION TREE



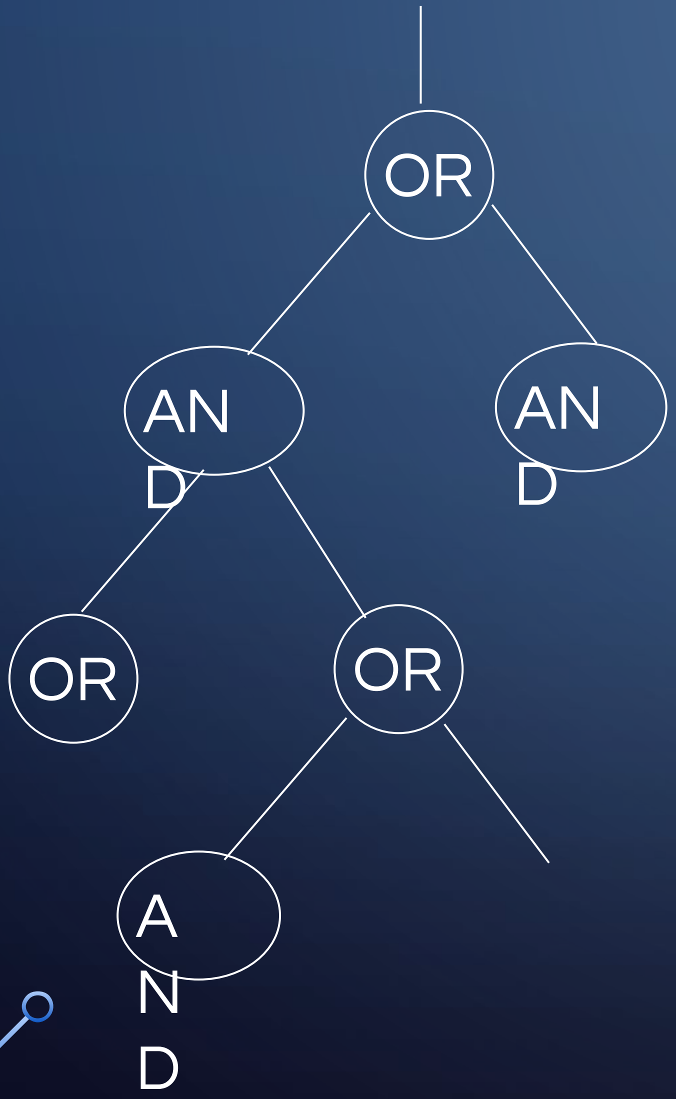
1st player

2nd player

- Position tree = formula;
- Output = YES if 1st player wins;

Unknown, irregular structure

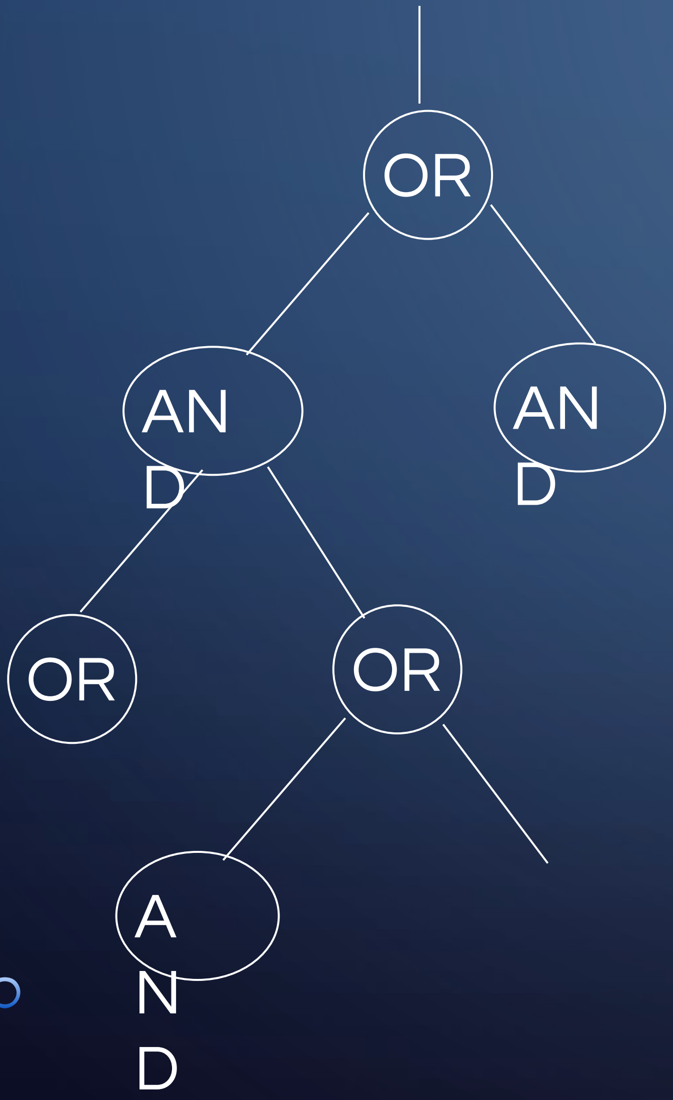
POSITION TREE



- AND/OR formula of size T can be evaluated by quantum algorithm in O(√T) steps [Reichardt, 2010].

- Model: structure of formula known, inputs unknown.

A, KOKAINIS, 2017



- AND-OR formula of unknown structure.

<u>Theorem</u> Formulas of size N and depth $N^{o(1)}$ can be evaluated in time $O(N^{1/2+o(1)})$.

SUMMARY (1)

- Quantum computers may be useful for
  - Simulating physics;
  - Factoring/codebreaking;
  - Search;

SUMMARY (2)

- Quantum advantage over many classical algorithms:
  - Simple exhaustive search;
  - Search by a random walk;
  - Nested search (formula evaluation);
  - Search on trees of unknown structure (backtracking).
- Usually, quadratic advantage but in very general settings.

CHALLENGES FOR FUTURE RESEARCH

- Speeding up dynamic programming?
- Quantum machine learning?
- Algorithms for first quantum computers (100 qubits, quite noisy)?