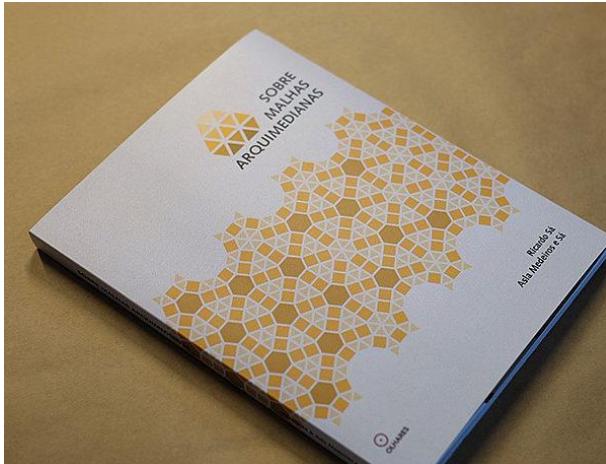




**SOBRE
MALHAS
ARQUIMEDIANAS**

LCG/UFRJ, 28 de março de 2018

O livro SOBRE MALHAS ARQUIMEDIANAS



Editora Olhares

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Ricardo Sá

Designer de produto

Diretor da AIRCRAFT GEOMETRY

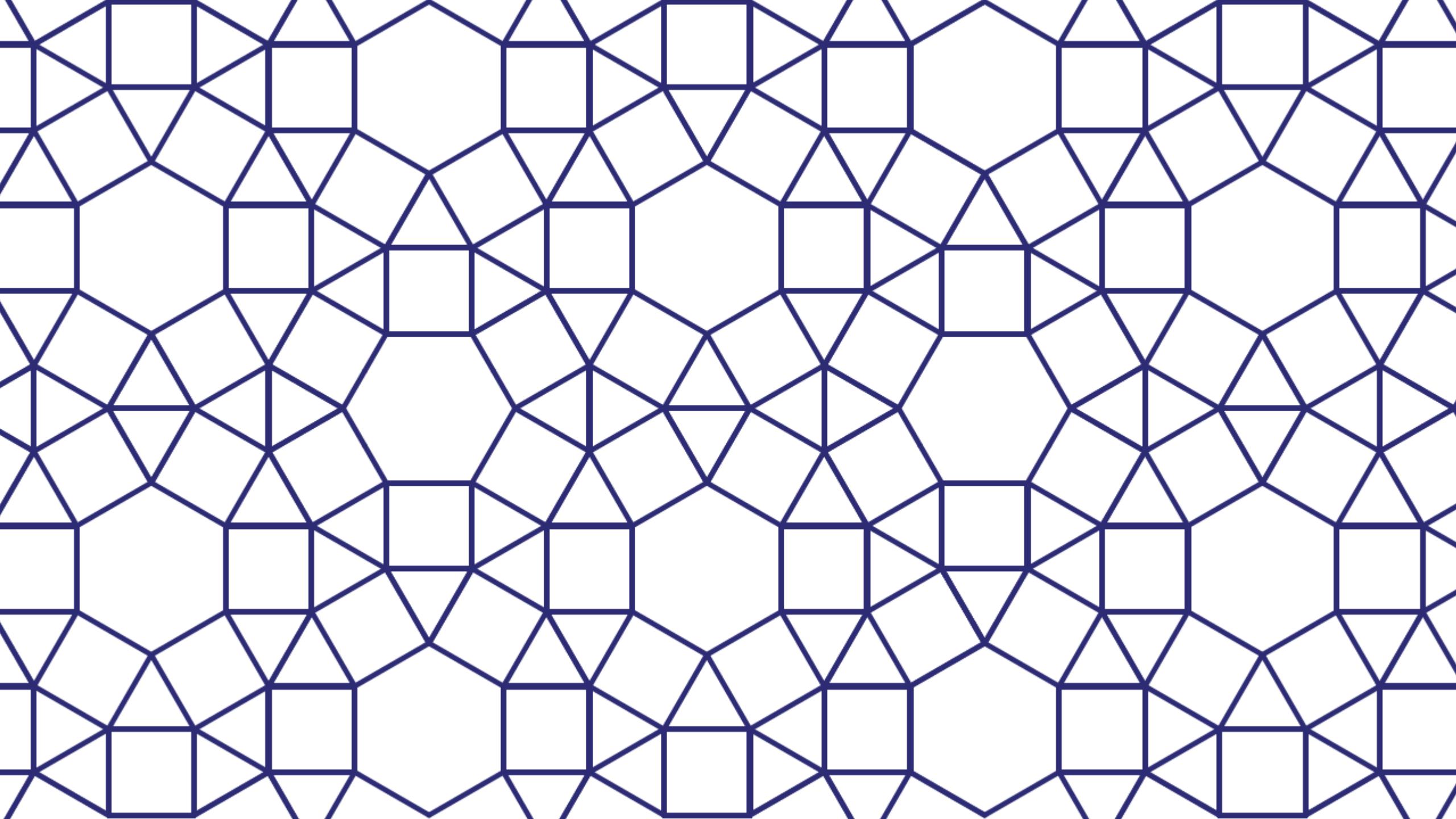


Asla Medeiros e Sá

Matemática com ênfase em Computação Gráfica

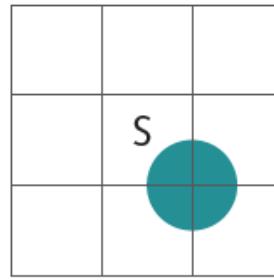
Professora da FGV/EMAp



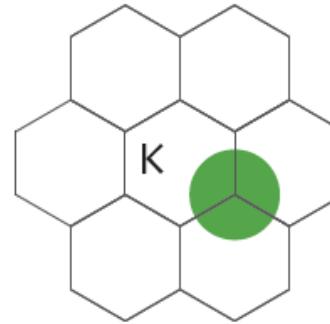
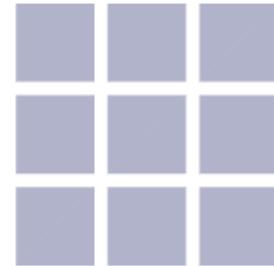


Capítulo 1

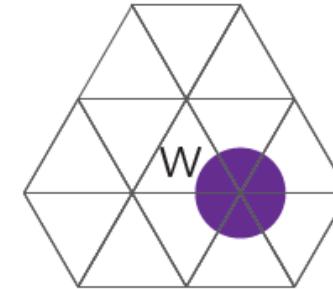
IDENTIFICAÇÃO DE MALHAS ARQUIMEDIANAS



#A{S}
S = 4-4-4-4



#A{K}
K = 6-6-6



#A{W}
W = 3-3-3-3-3-3



Nós Arquimedianos

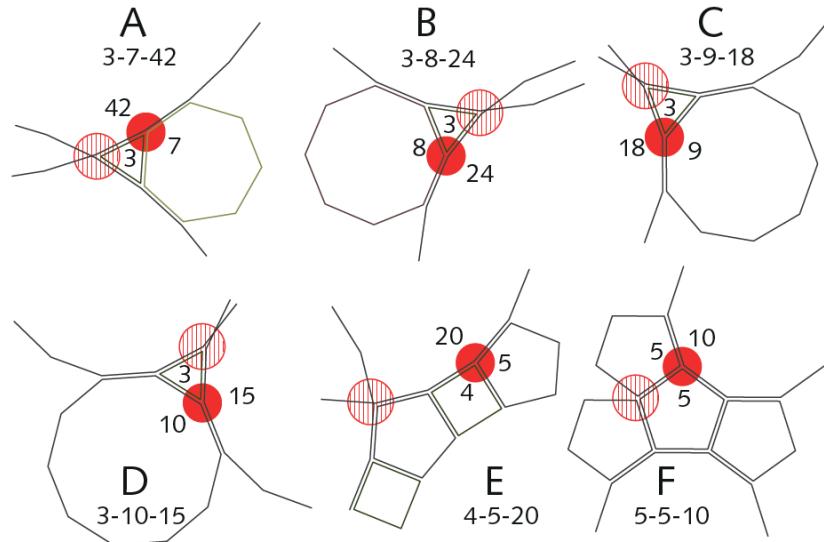
Arranjos de polígonos que formam tesselações arquimedianas obedecem a uma regra de formação simples: só podemos encaixar grupos de polígonos em torno de um vértice quando os ângulos internos dos polígonos que compartilham o vértice somem 360° .

Lembrando que o ângulo interno (ângulo entre duas arestas consecutivas) de um polígono regular convexo de n lados é dado por $\frac{(n-2)180^\circ}{n}$ podemos observar que:

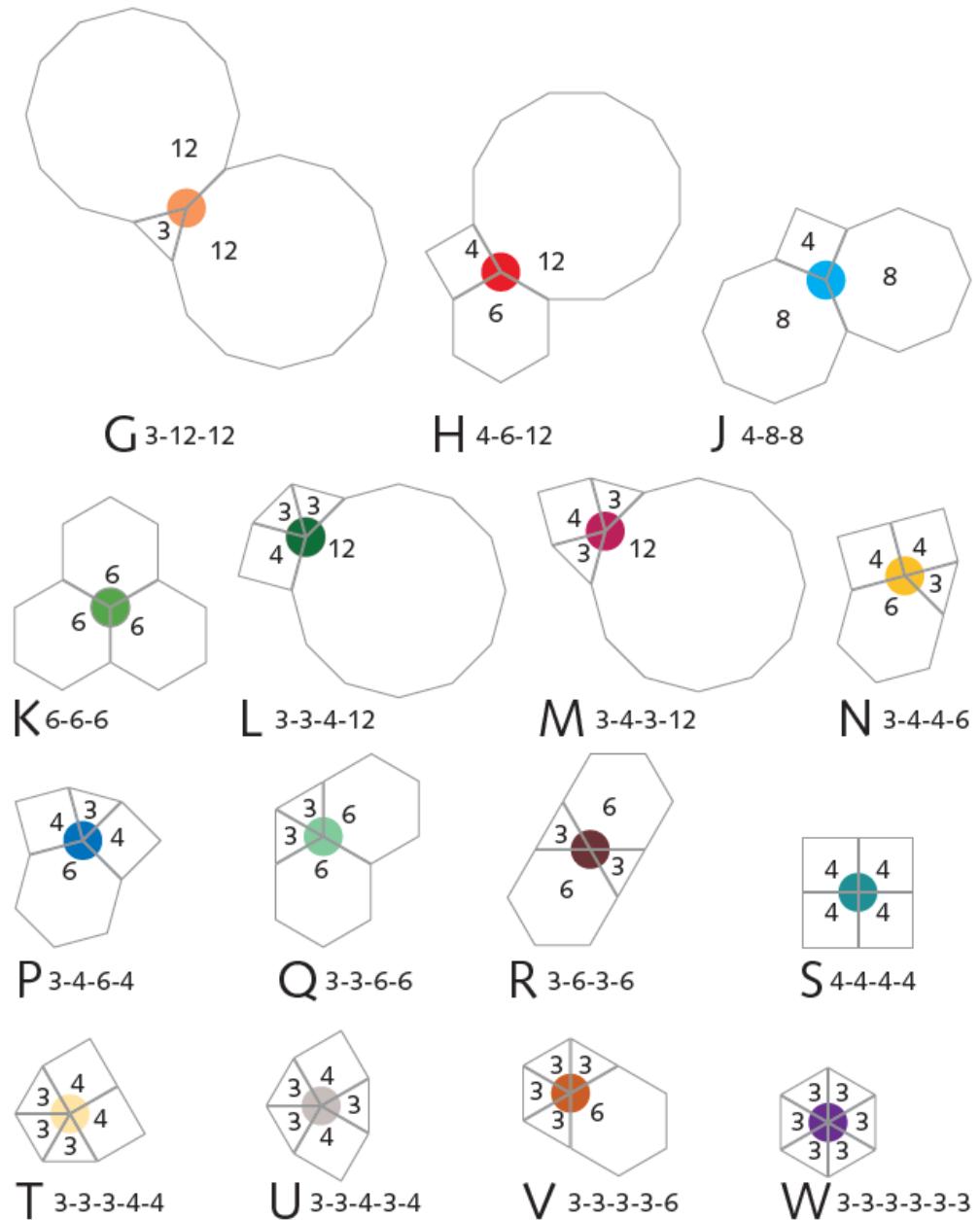
- O ângulo interno de um polígono regular convexo é menor que 180° . Portanto, o menor número de polígonos em torno de um vértice é 3.
- O menor ângulo interno de um polígono regular convexo é 60° , para $n=3$ (triângulo). Portanto, o maior número de polígonos em torno de um vértice será de 6 triângulos.

NÓ	N1	N2	N3	N4	N5	N6
A	3	7	42	-	-	-
B	3	8	24	-	-	-
C	3	9	18	-	-	-
D	3	10	15	-	-	-
E	4	5	20	-	-	-
F	5	5	10	-	-	-
G	3	12	12	-	-	-
H	4	6	12	-	-	-
J	4	8	8	-	-	-
K	6	6	6	-	-	-
L	3	3	4	12	-	-
M	3	4	3	12	-	-
N	3	4	4	6	-	-
P	3	4	6	4	-	-
Q	3	3	6	6	-	-
R	3	6	3	6	-	-
S	4	4	4	4	-	-
T	3	3	3	4	4	-
U	3	3	4	3	4	-
V	3	3	3	3	6	-
W	3	3	3	3	3	3

Nós Arquimedianos

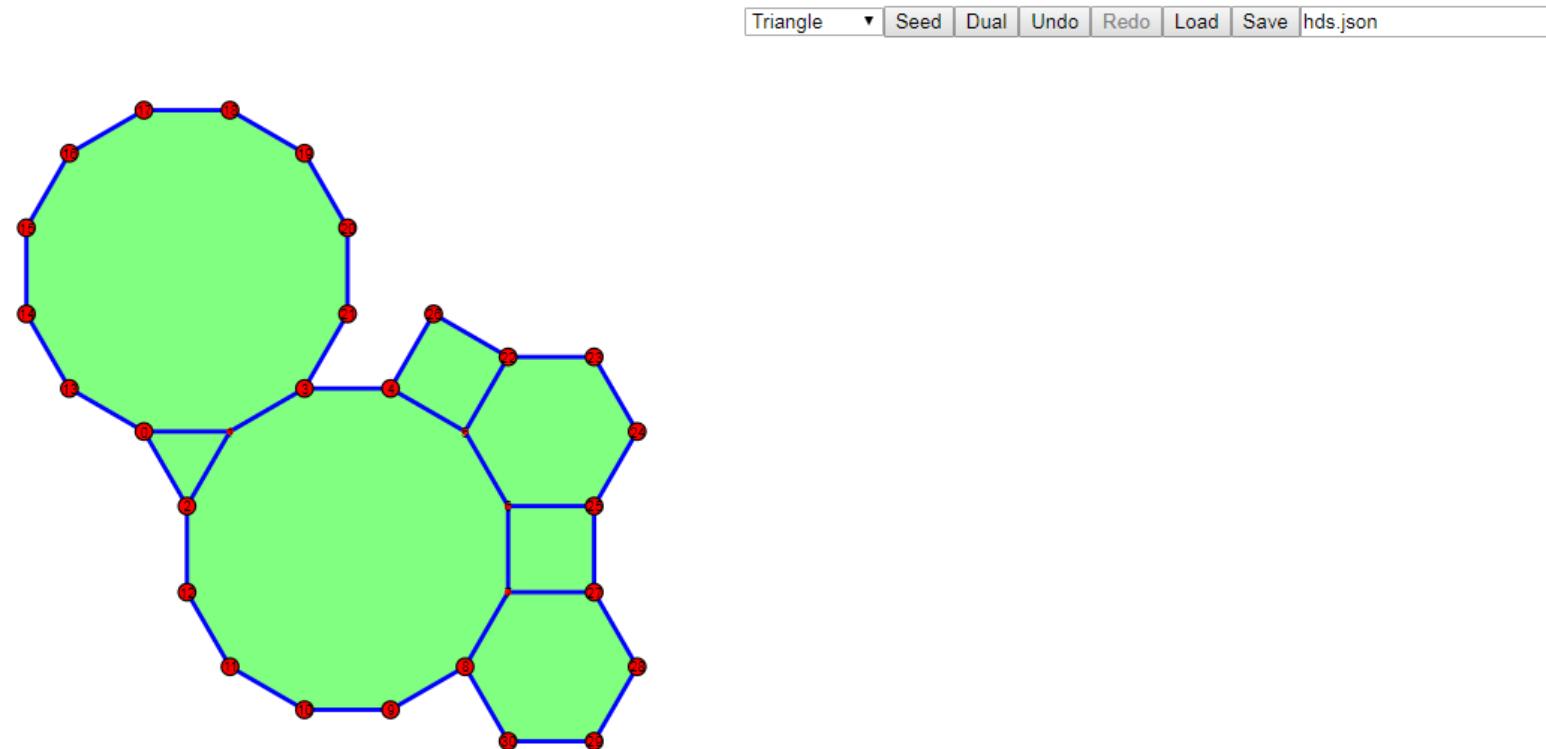


	TRIÂNGULO
	QUADRADO
	HEXÁGONO
	OCTÓGONO
	DODECÁGONO

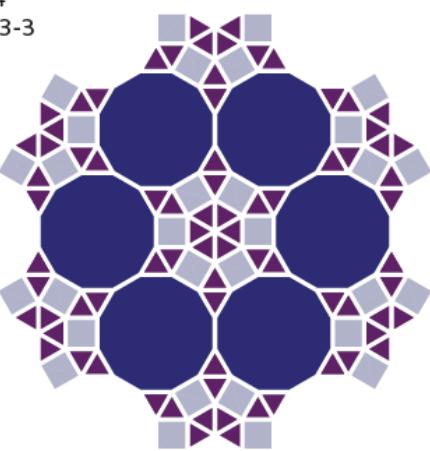
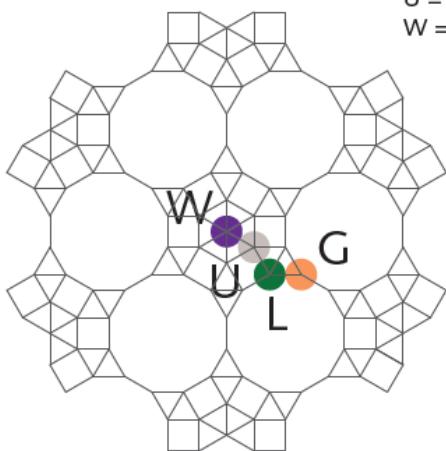
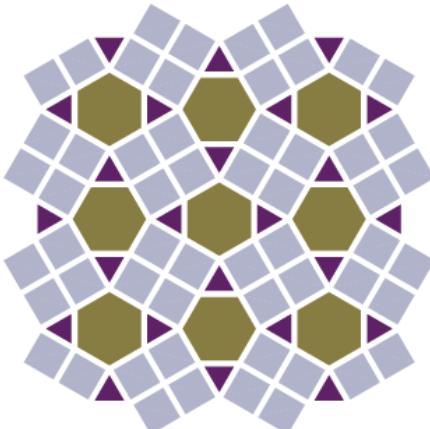
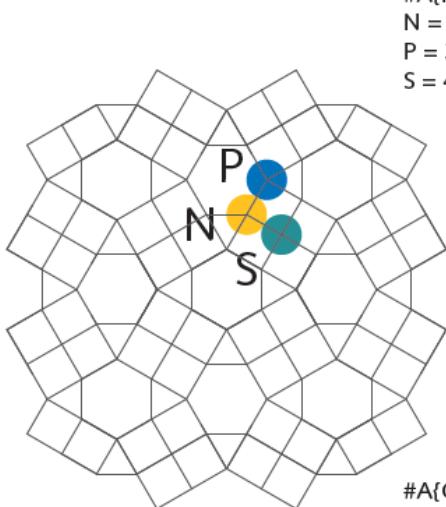


Combinação de nós

- <http://cesperanca.org/archimedean/>
- <https://www.youtube.com/watch?v=c4PD1T1I9u8&feature=youtu.be>

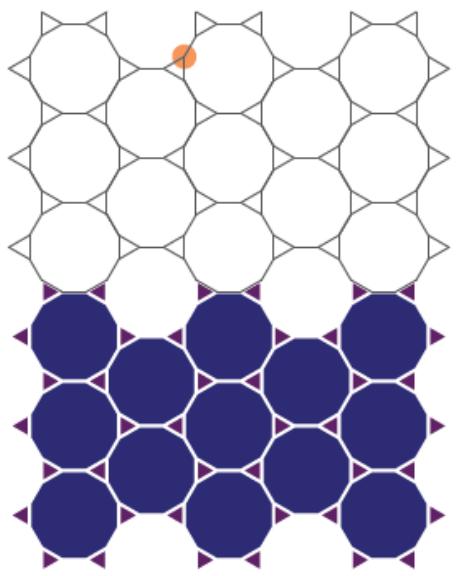


Malhas Arquimedianas #A{X,Y,...}[i]

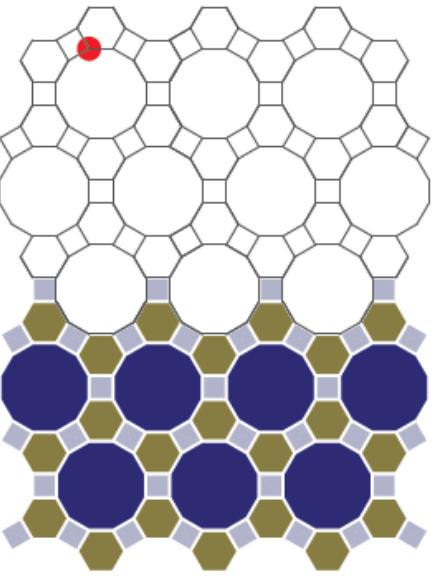


NÓ	N1	N2	N3	N4	N5	N6
A	3	7	42	-	-	-
B	3	8	24	-	-	-
C	3	9	18	-	-	-
D	3	10	15	-	-	-
E	4	5	20	-	-	-
F	5	5	10	-	-	-
G	3	12	12	-	-	-
H	4	6	12	-	-	-
J	4	8	8	-	-	-
K	6	6	6	-	-	-
L	3	3	4	12	-	-
M	3	4	3	12	-	-
N	3	4	4	6	-	-
P	3	4	6	4	-	-
Q	3	3	6	6	-	-
R	3	6	3	6	-	-
S	4	4	4	4	-	-
T	3	3	3	4	4	-
U	3	3	4	3	4	-
V	3	3	3	3	6	-
W	3	3	3	3	3	3

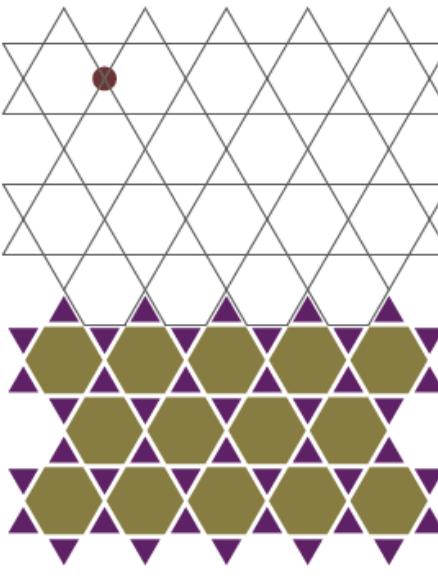
- ▲ TRIÂNGULO
- QUADRADO
- ◆ HEXÁGONO
- OCTÓGONO
- DODECÁGONO



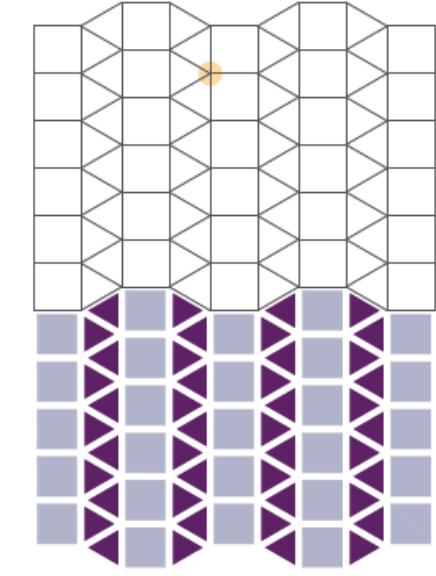
$\#A_{\{G\}}$



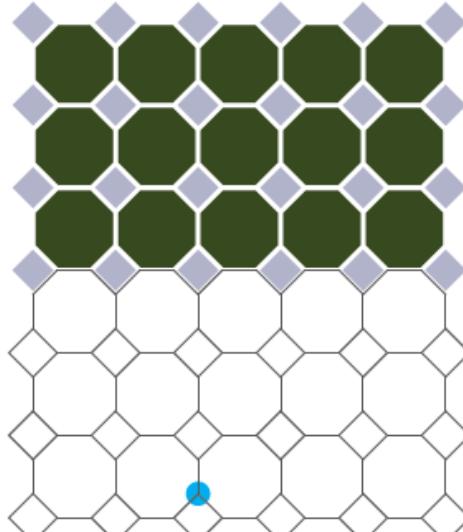
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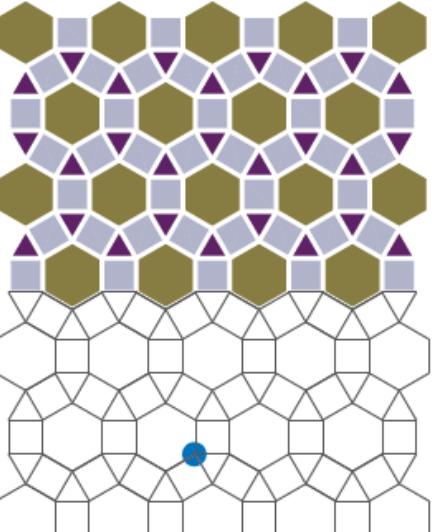
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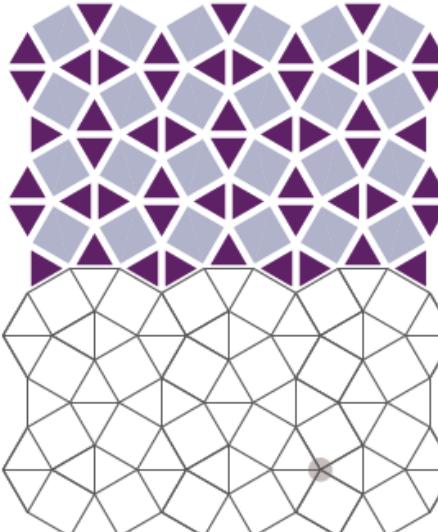
$\#A_{\{T\}}$



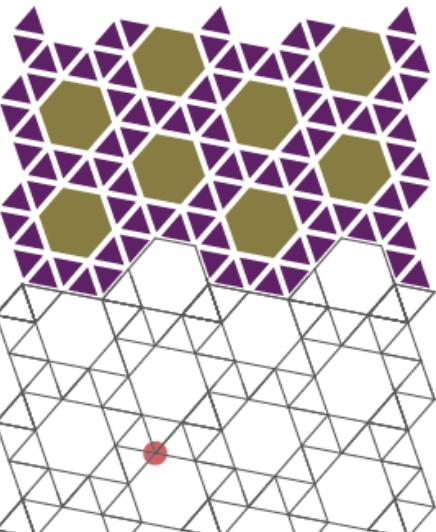
$\#A_{\{J\}}$



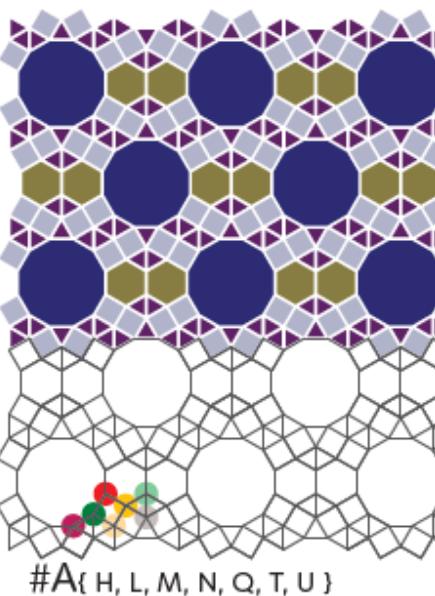
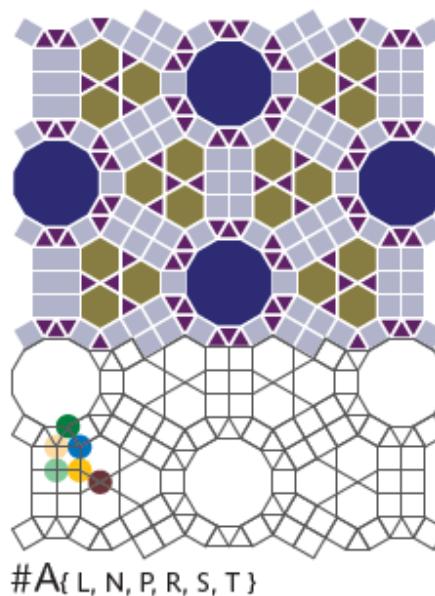
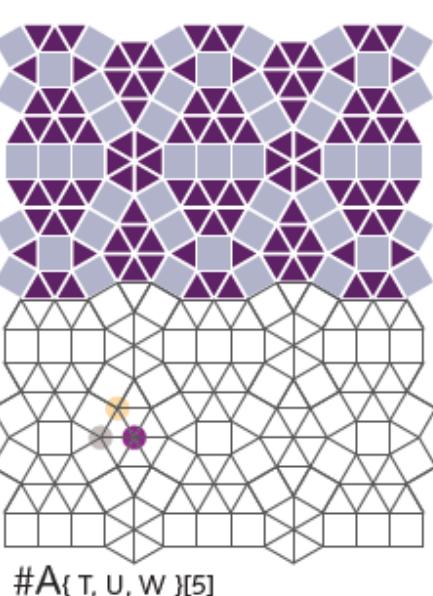
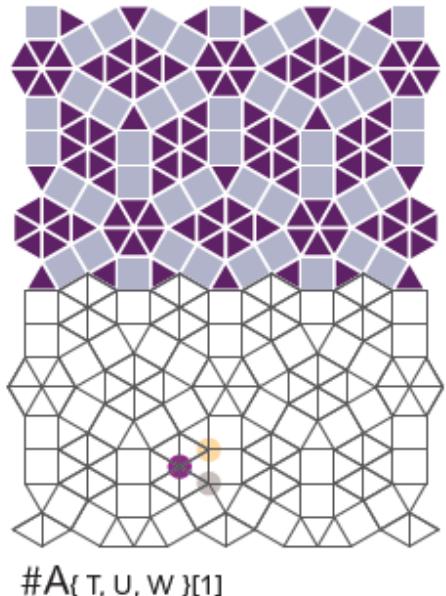
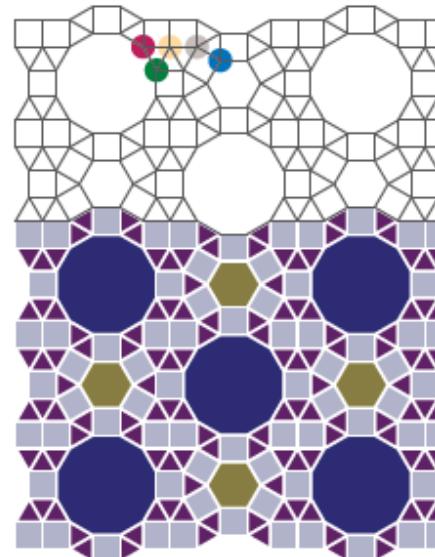
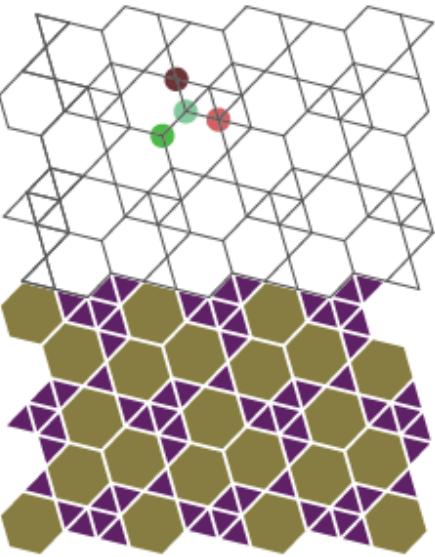
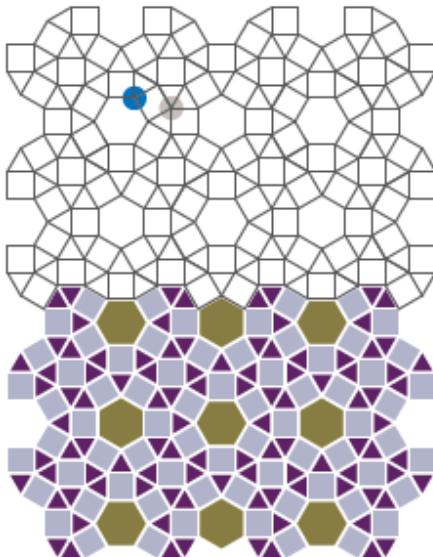
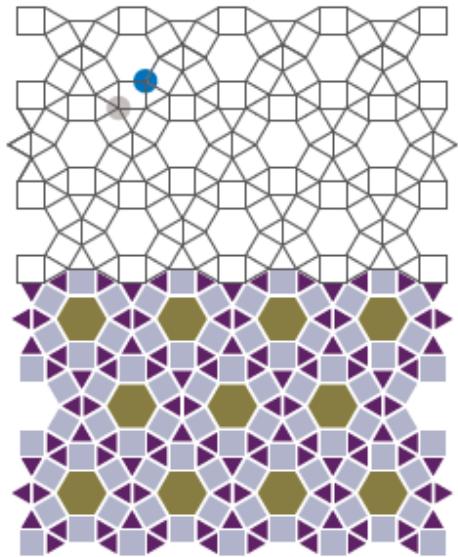
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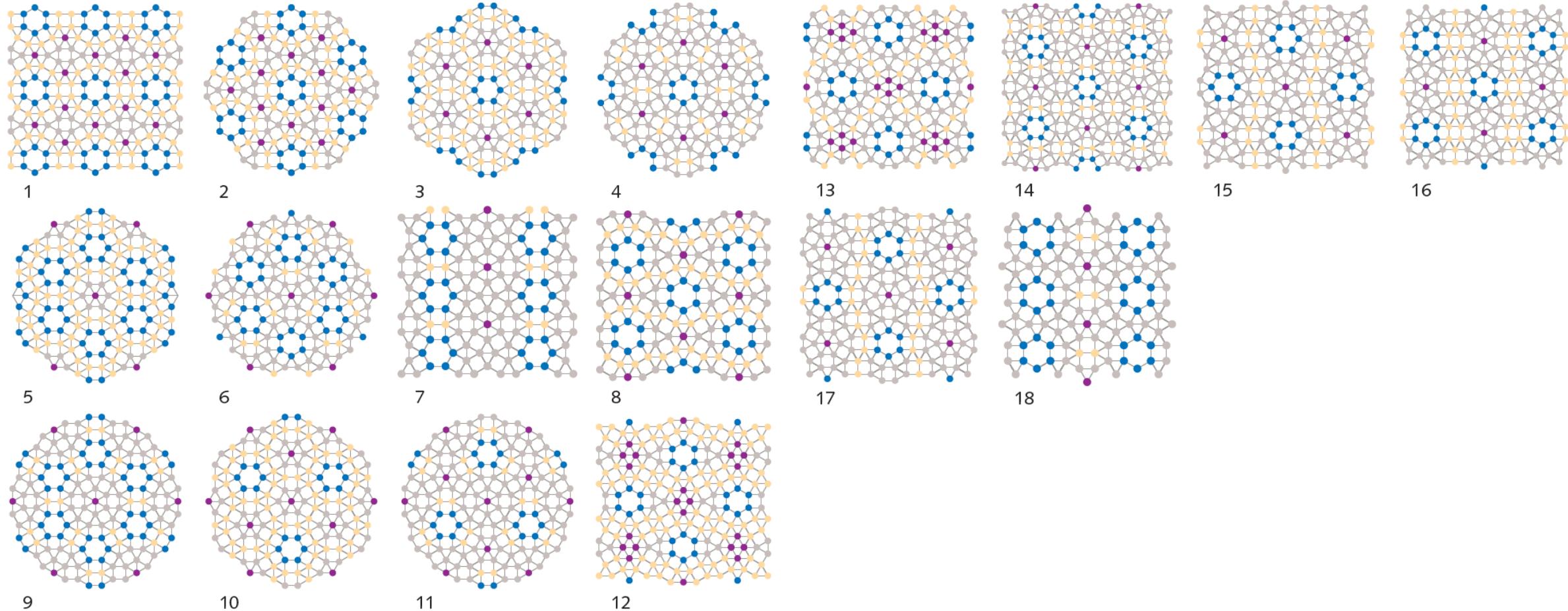
$\#A_{\{U\}}$



$\#A_{\{V\}}$



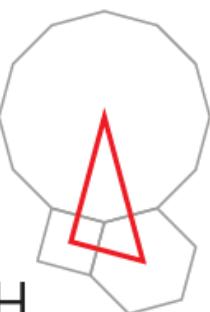
$\#A\{P,T,U,W\}[i]$



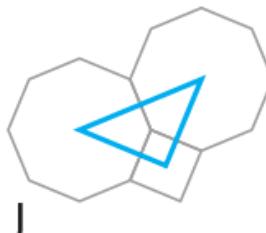
Polígonos Duais



G



H



J



K



L



M



N



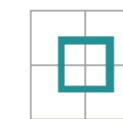
P



Q



R



S



T



U

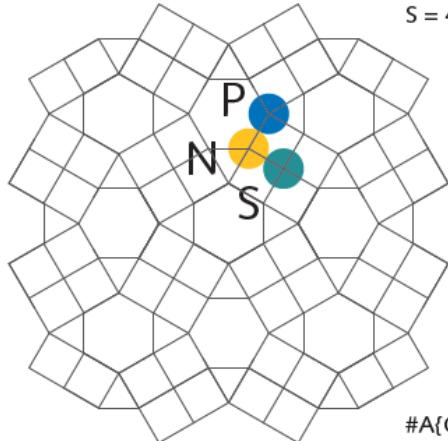


V

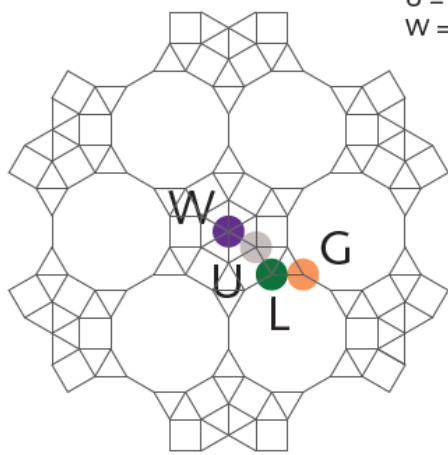
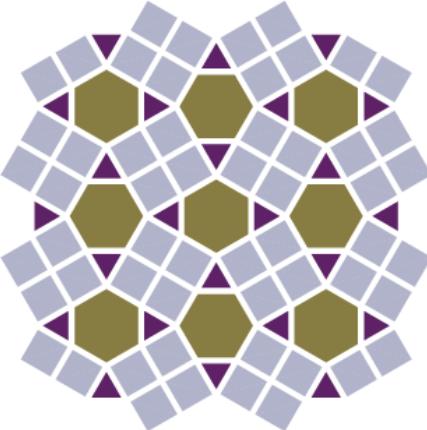


W

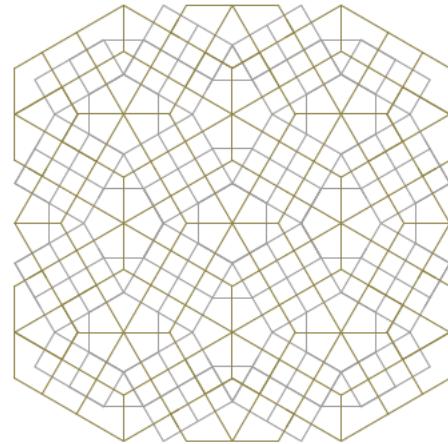
Malhas Duais #D{X,Y,...}[i]



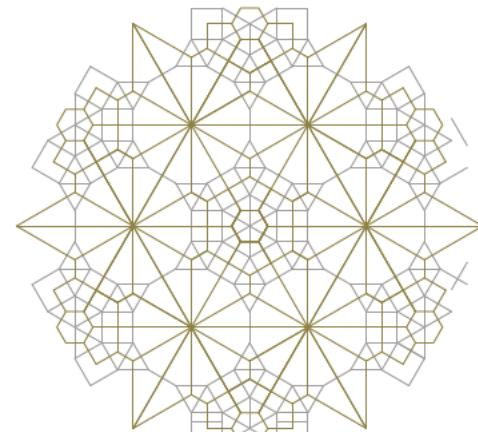
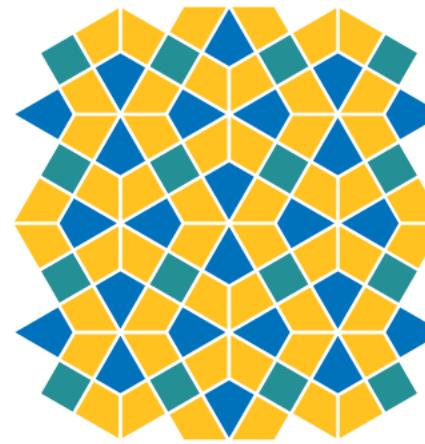
#A{N,P,S}
N = 3-4-4-6
P = 3-4-6-4
S = 4-4-4-4



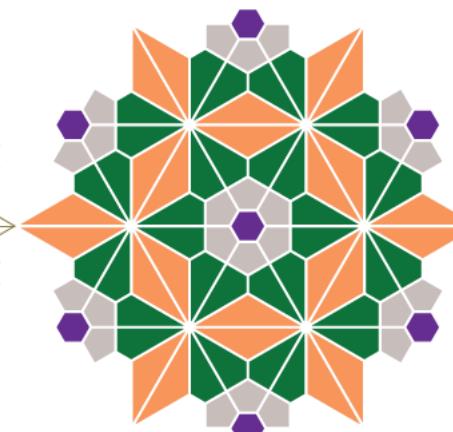
#A{G,L,U,W}
G = 3-12-12
L = 3-3-4-12
U = 3-3-4-3-4
W = 3-3-3-3-3-3



#D{N,P,S}

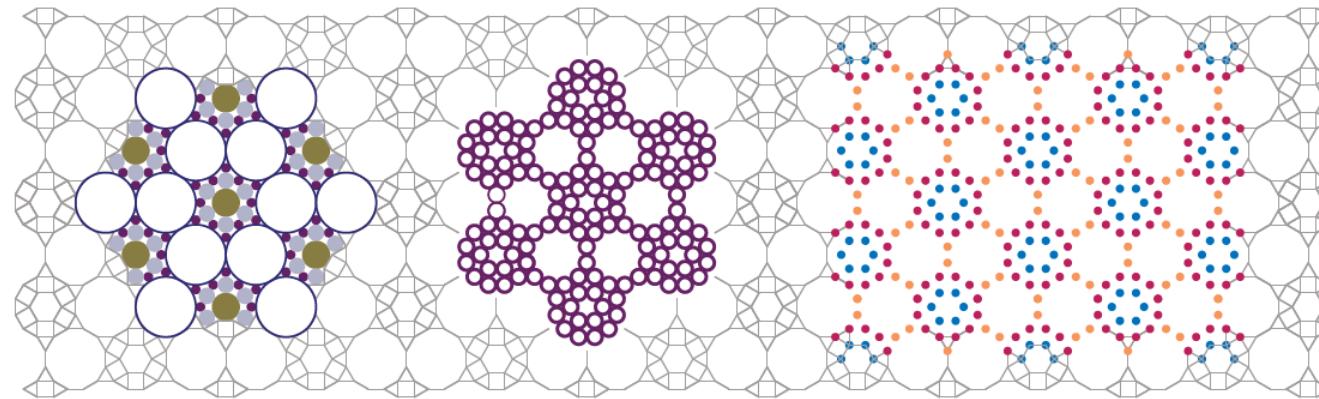
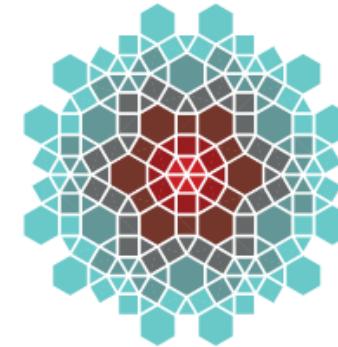
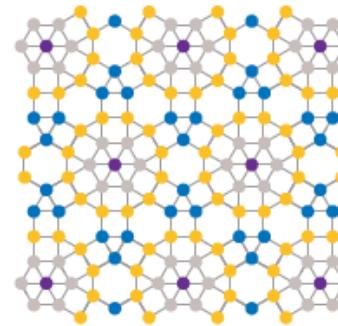


#D{G,L,U,W}

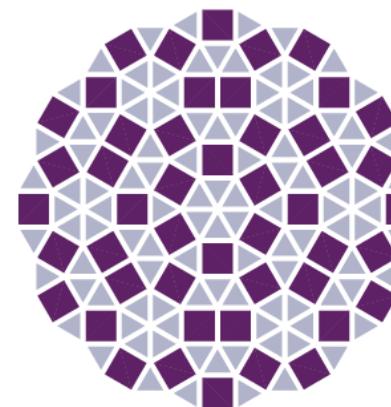
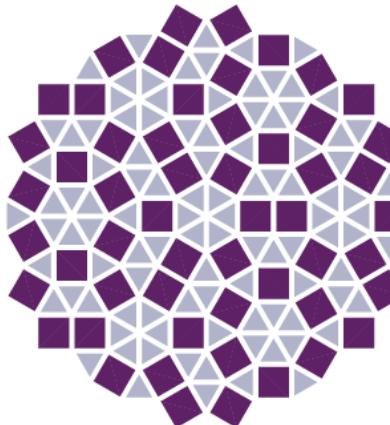
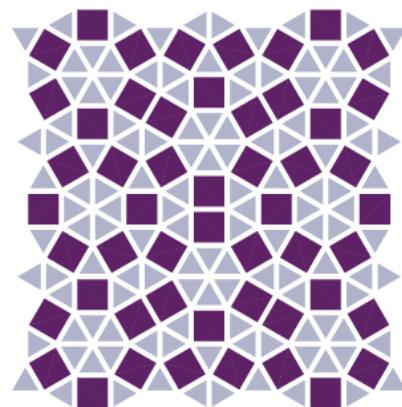
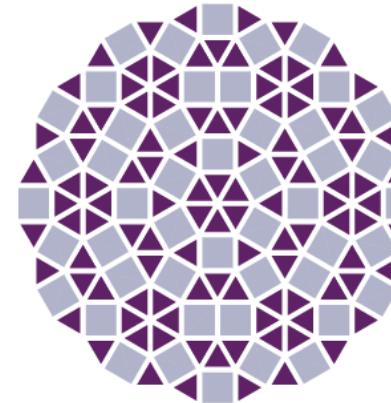
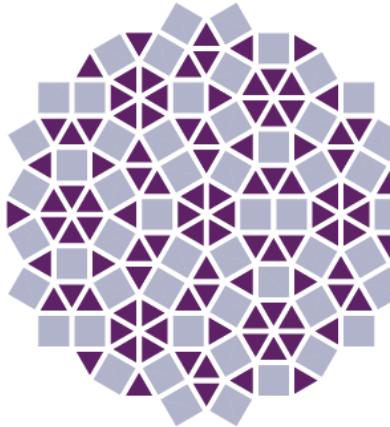
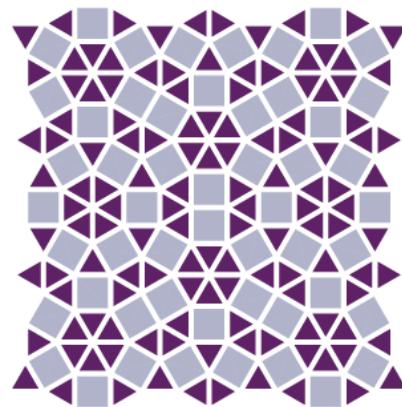


Capítulo 2

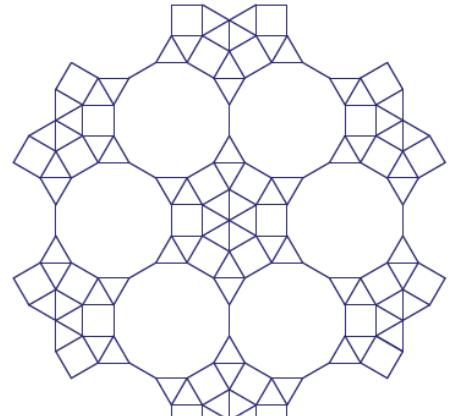
MODALIDADES GRÁFICAS



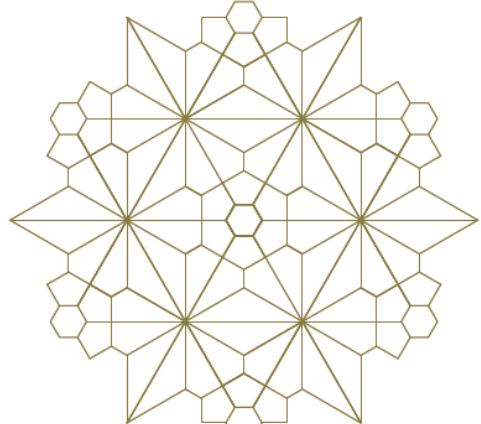
Grupamentos de tesselas



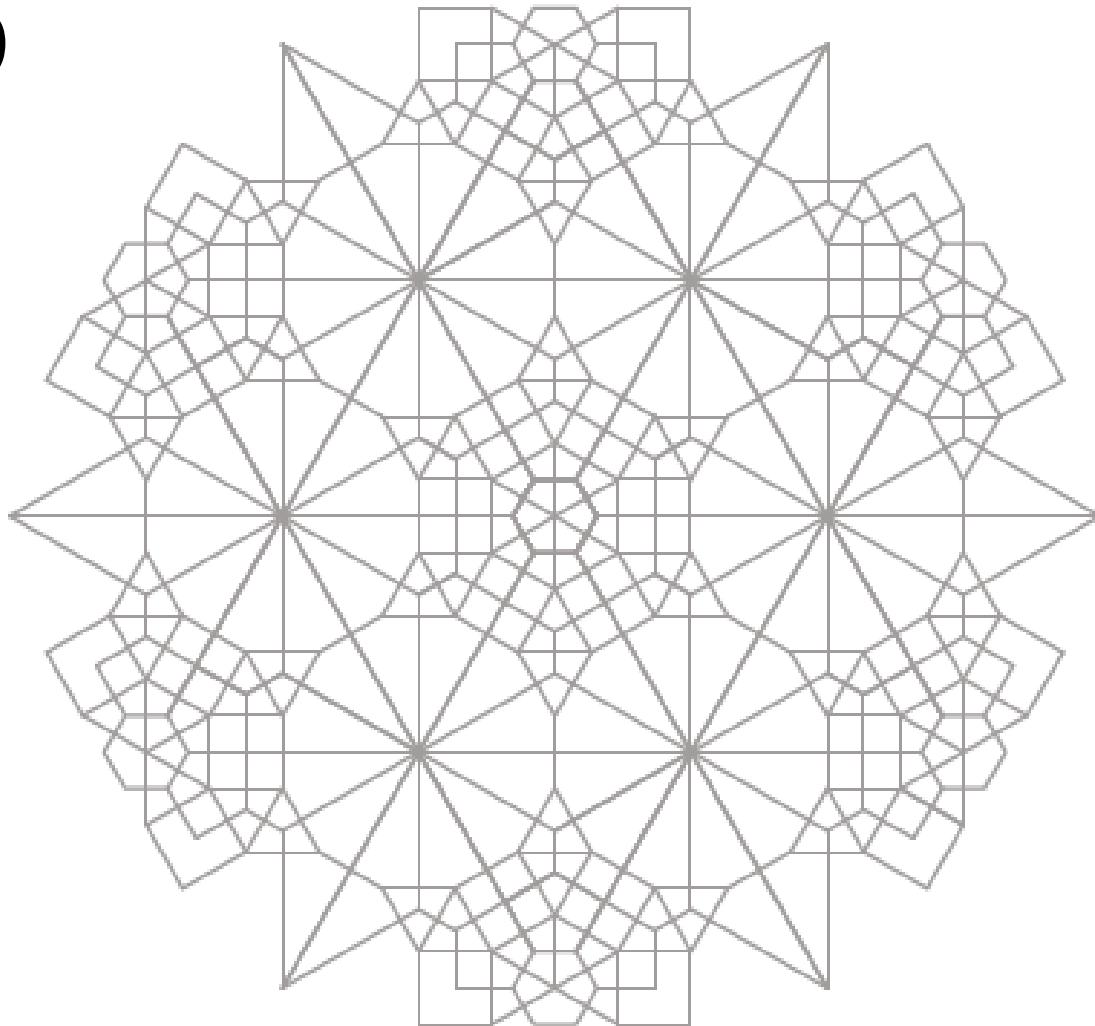
Combinação #A + #D



#A{ G,L,U,W }

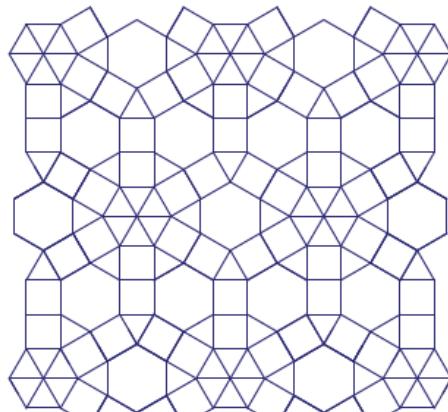


#D{ G,L,U,W }

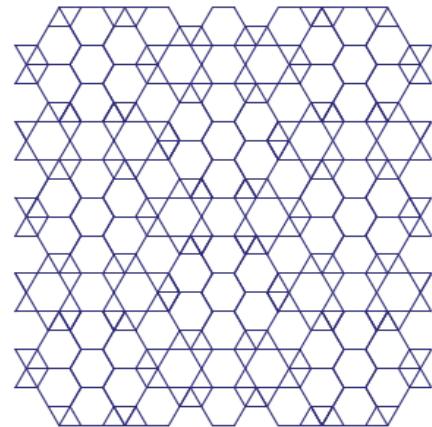


#A+D{ G,L,U,W }

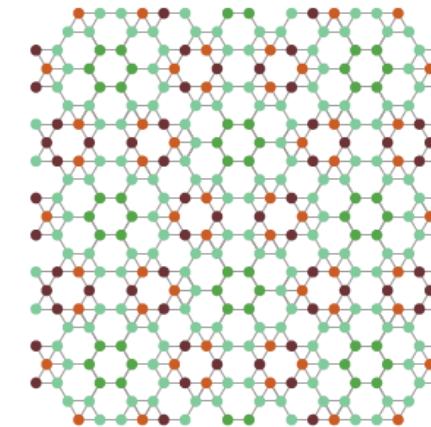
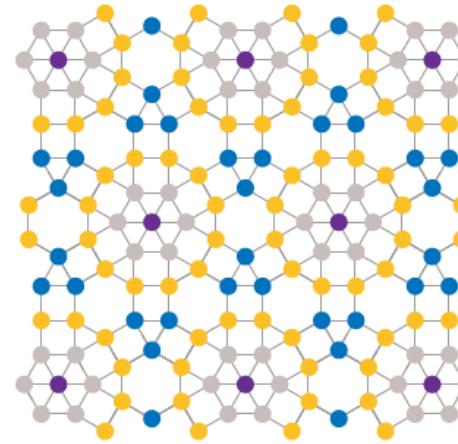
Constelação de nós



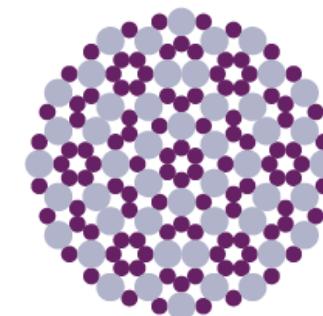
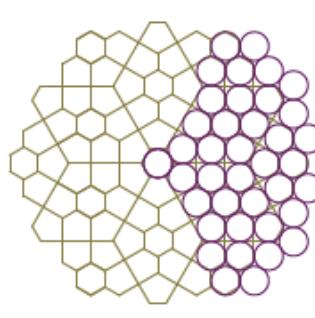
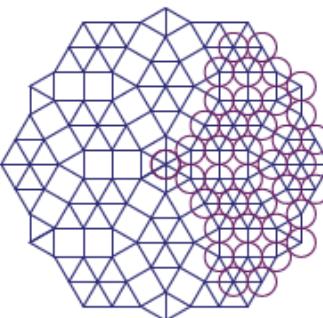
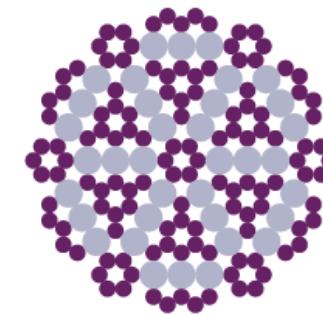
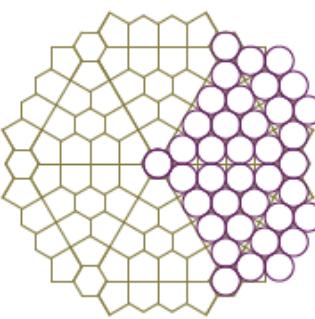
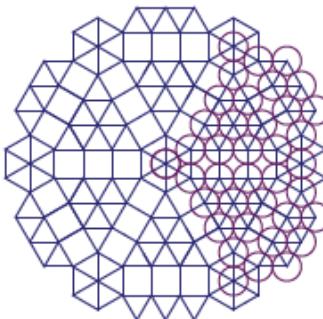
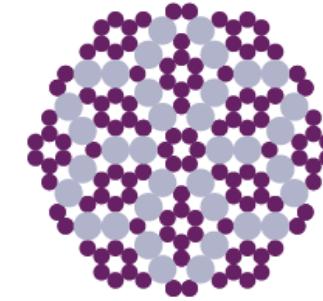
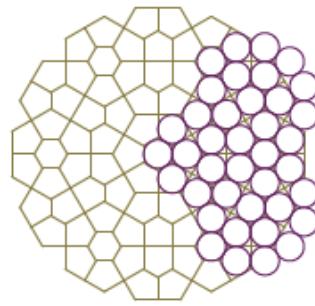
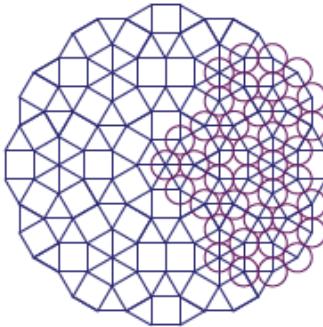
$\#A_{\{N,P,U,W\}}$

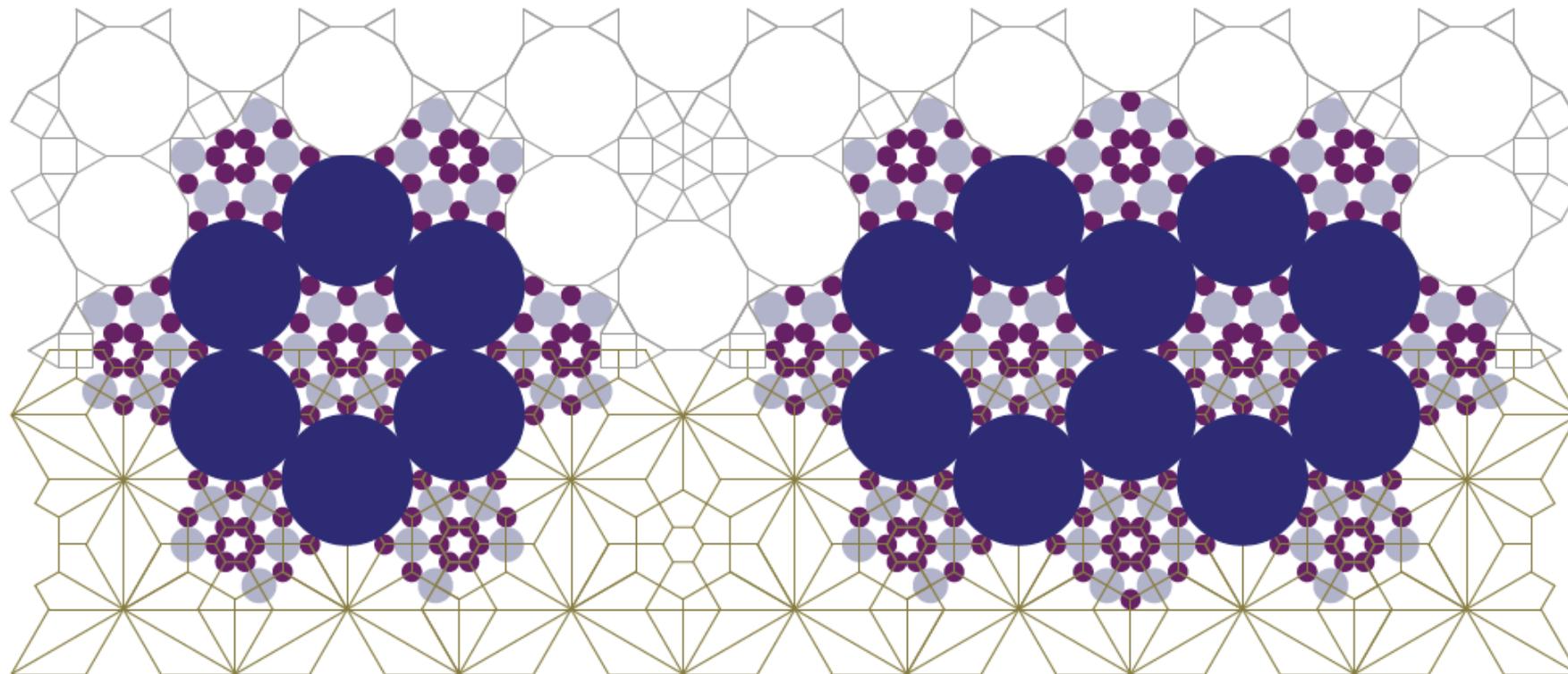
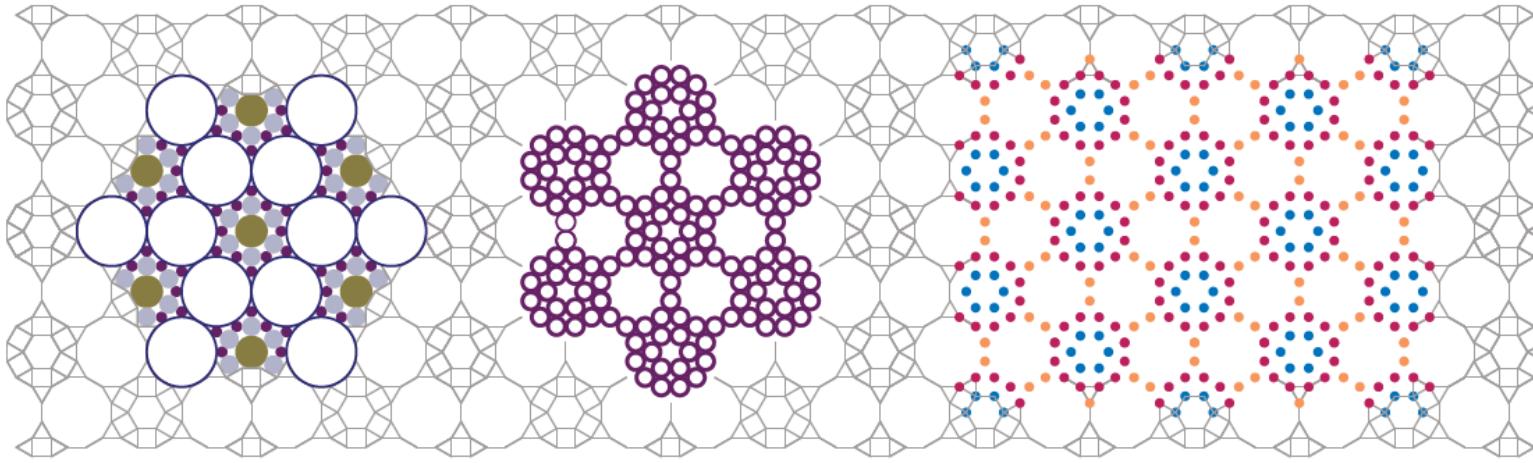


$\#A_{\{K,Q,R,V\}}[4]$



Arranjos de círculos





Crescimento por linhas de fratura

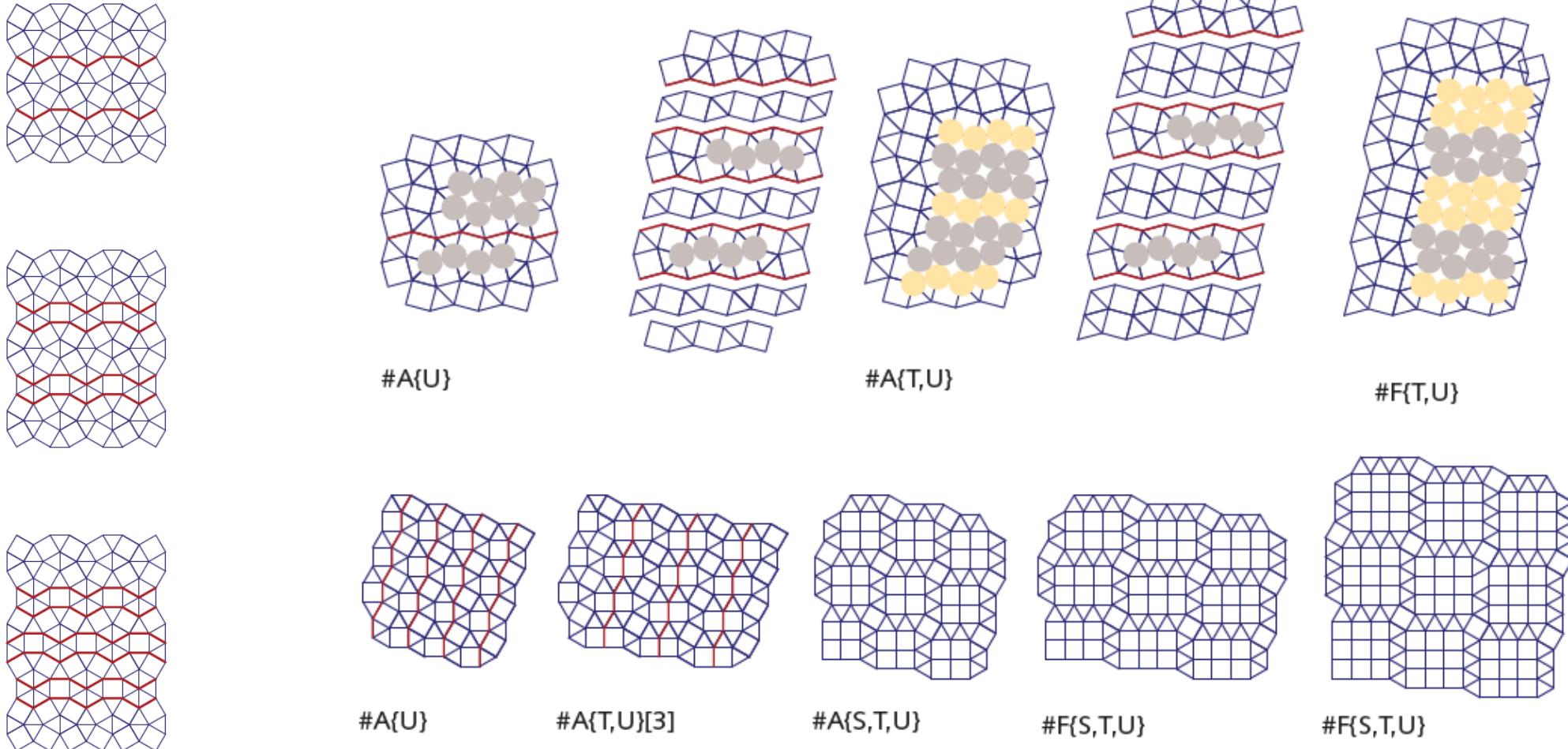
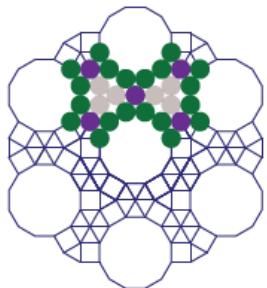
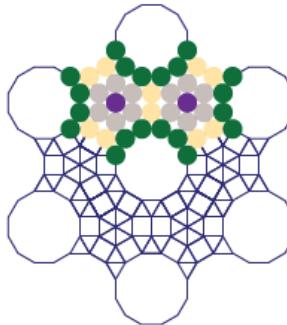


Figura 2.21 – Exemplo de crescimento de malha por inserção de frisos a partir da malha $\#A\{U\}$.

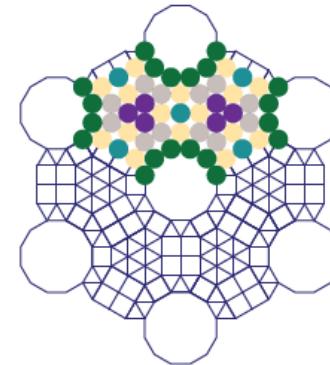
Crescimento radial



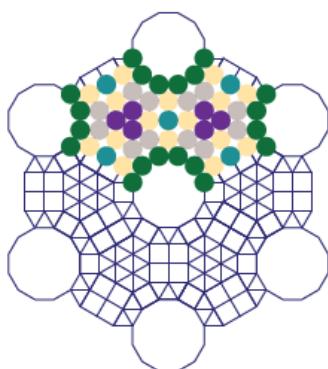
#A{L,U,W}



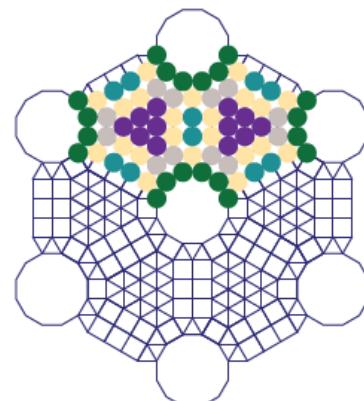
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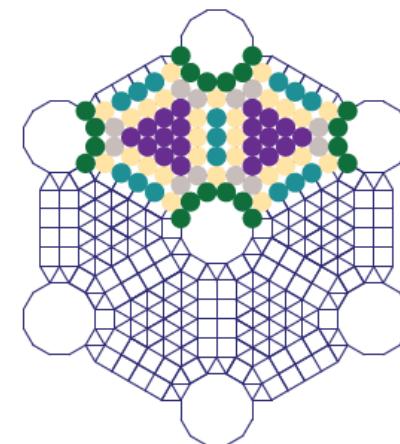
#A{L,S,T,U,W}



#A{L,S,T,U,W}



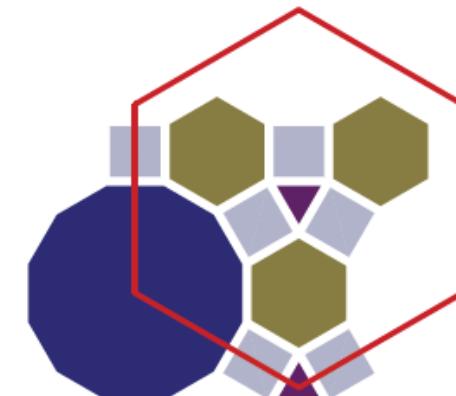
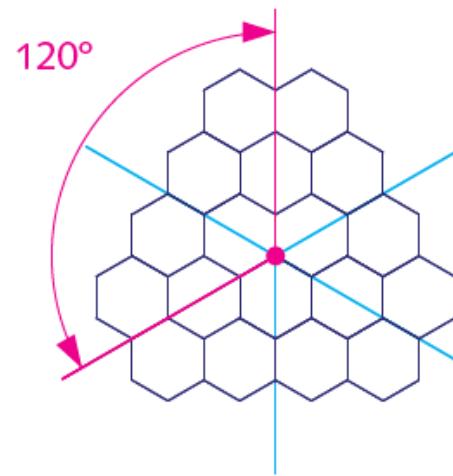
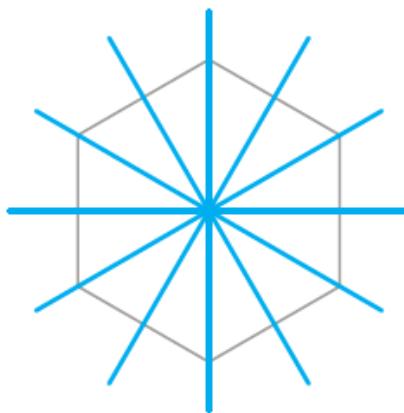
#R{L,S,T,U,W}



#R{L,S,T,U,W}

Capítulo 3

SIMETRIAS EM MALHAS ARQUIMEDIANAS



Análise de simetrias

CONWELL	CRISTALOGRAFIA
** 2 eixos de reflexão paralelos	pm
*2222 4 pares de eixos de reflexão	pmm
*333 3 conjuntos de 3 eixos de reflexão	p3m1
*442 2 conjuntos de 4 e 1 conjunto de 2 eixos de reflexão	p4m
*632 1 conjunto de 6, 1 de 3 e 1 de 2 eixos de reflexão	p6m
2222 4 centros de rotação de 180°	p2
333 3 centros de rotação de 120°	p3
442 2 centros de rotação de 90° e 1 de 180°	p4
632 1 centro de rotação de 60°, 1 de 120° e 1 de 180°	p6
2*22 rotação de 180° com 2 pares de eixos de reflexão	cmm
4*2 rotação de 90° com 1 par de eixos de reflexão	p4g
3*3 rotação de 120° e 1 conjunto de três eixos de reflexão	p31m
22* 2 centros de rotação de 180° com 1 eixo de reflexão	pmg
*x 1 eixo de reflexão e 1 de reflexão com deslizamento	cm
xx 2 eixos de reflexão com deslizamento	pg
22x 2 centros de rotação 180° e 1 de reflexão com deslizamento	pgg
0 apenas translação	p1

Tabela 3.1 – Os 17 tipos de grupos de simetria também conhecidos como grupos cristalográficos. Na coluna da esquerda apresentamos a nomenclatura introduzida por Conwell. Na coluna da direita vemos a correspondente nomenclatura utilizada em cristalografia.

UNIDADE TRANSLACIONAL

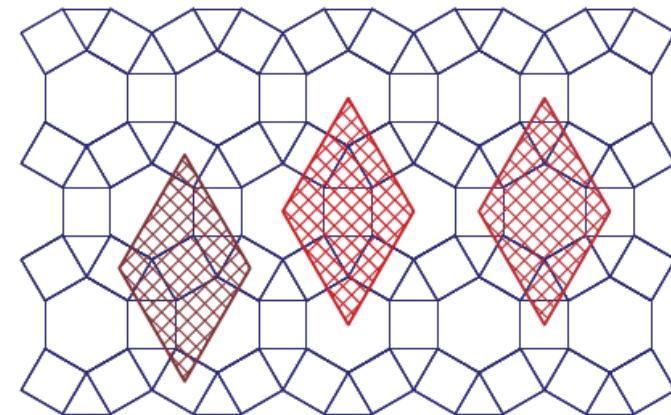
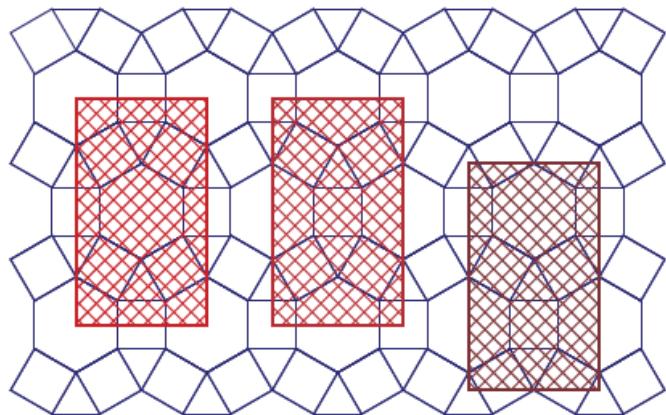
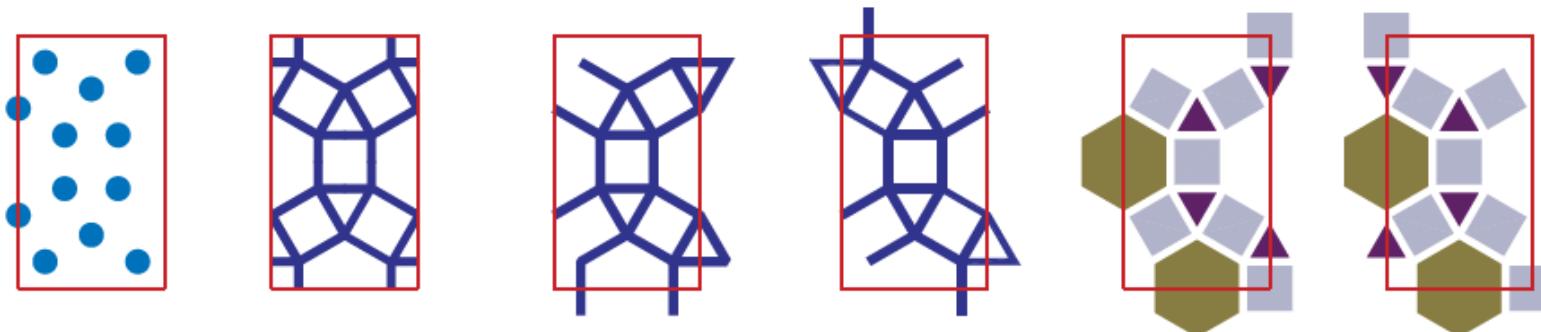
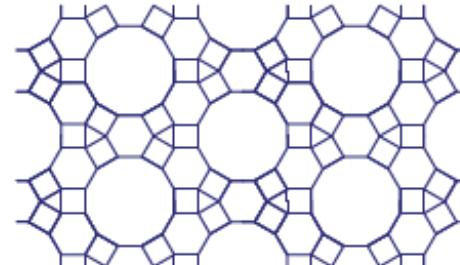
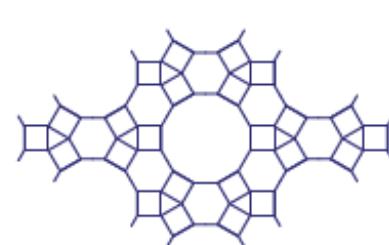
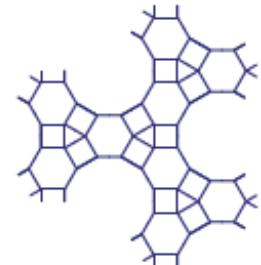
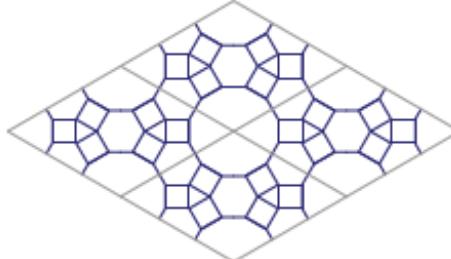
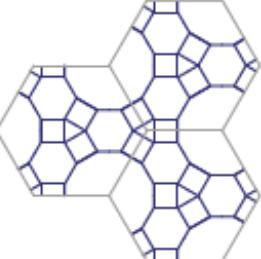
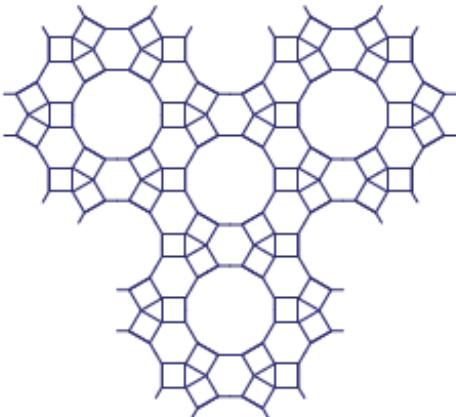
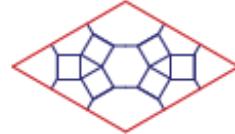
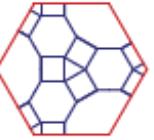
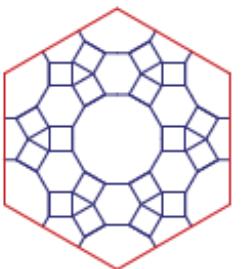
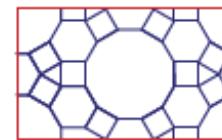
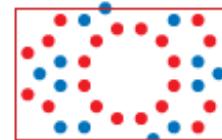
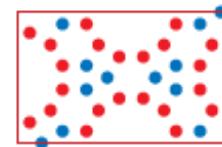
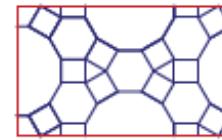
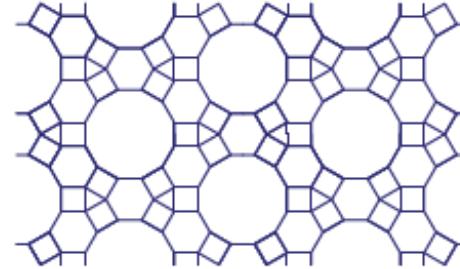
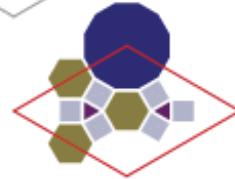
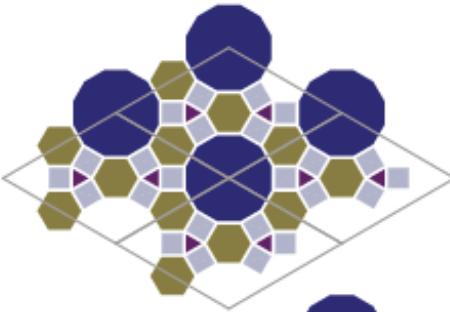
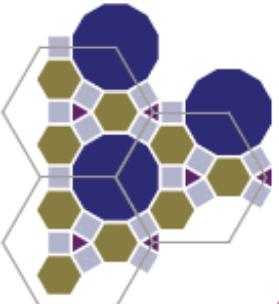
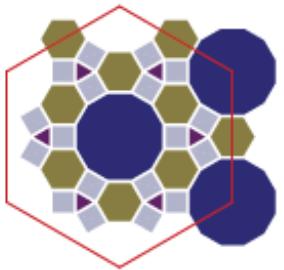
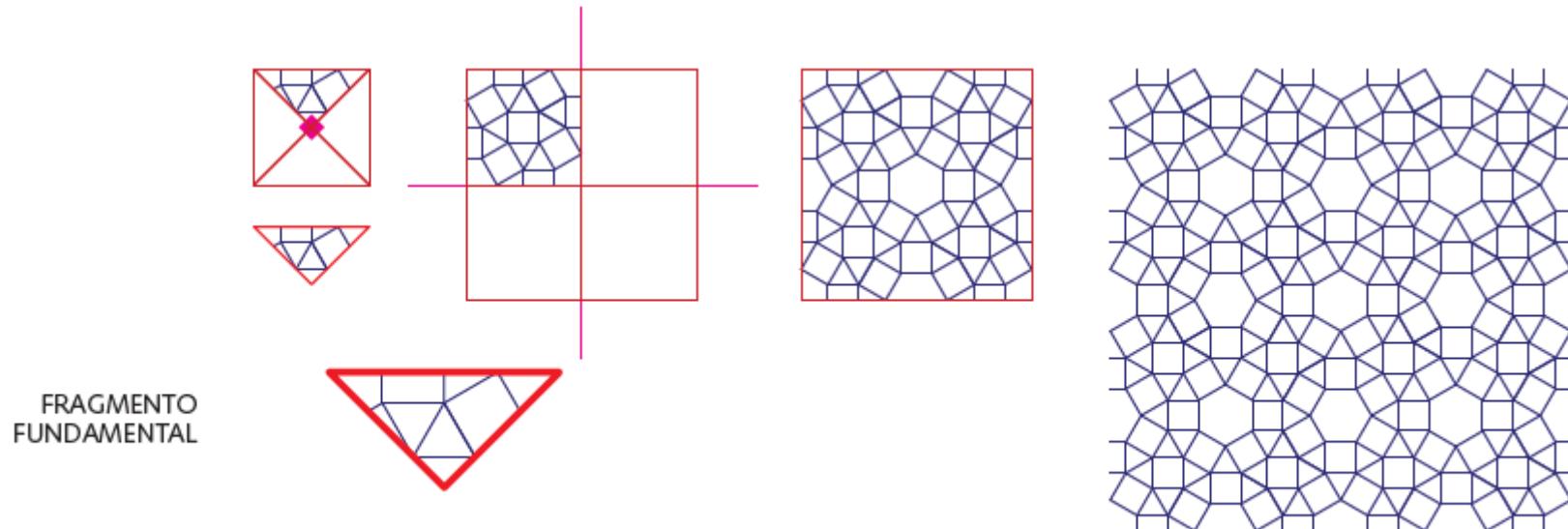
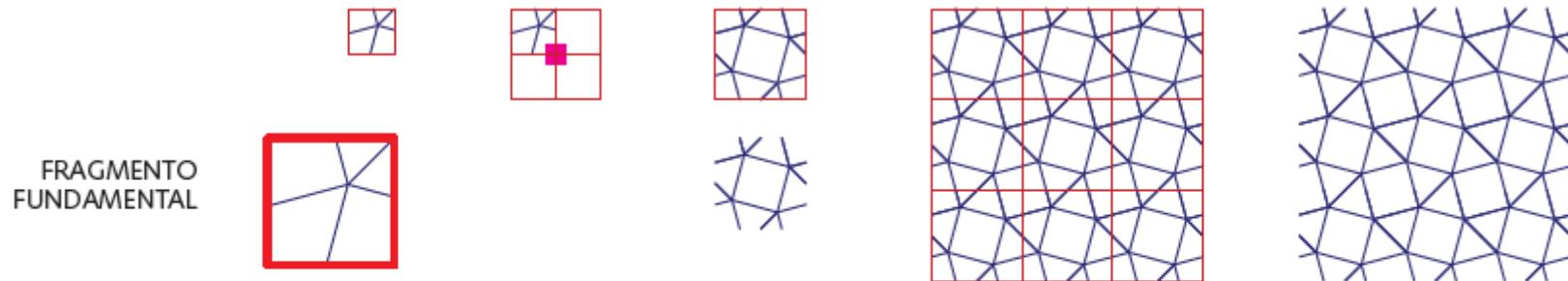


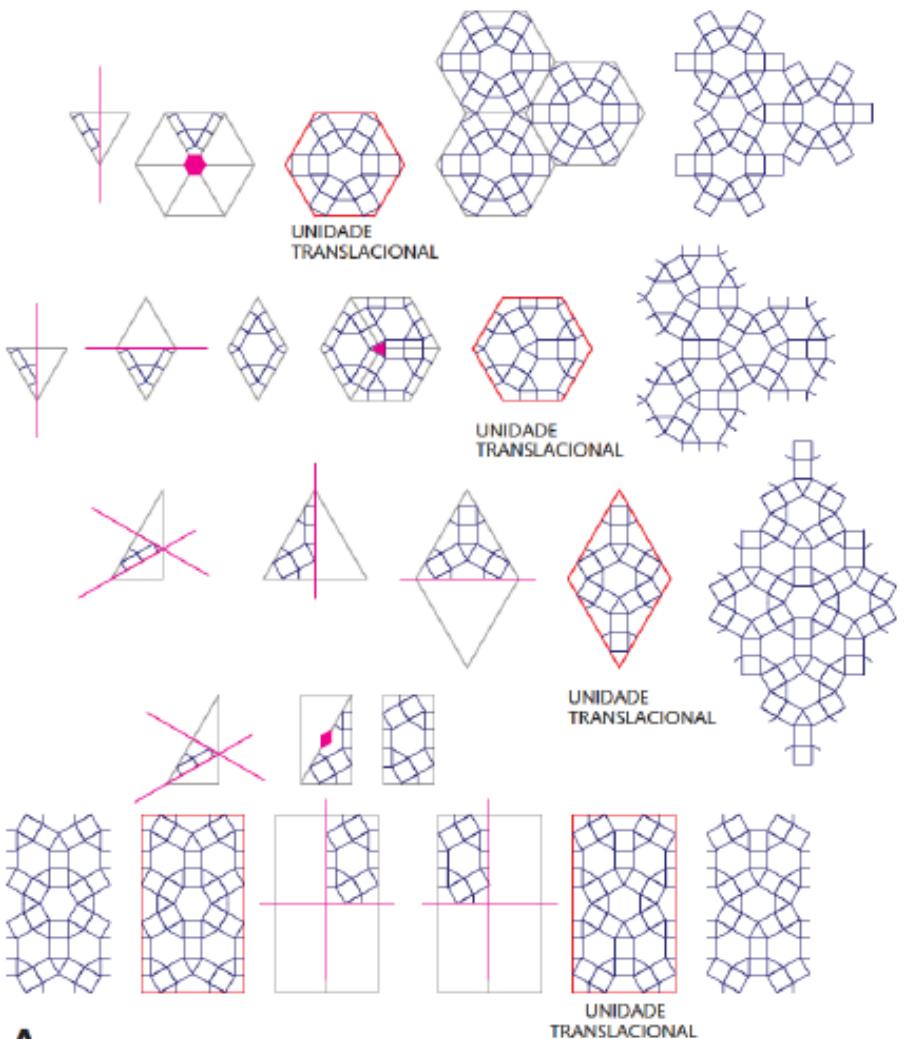
Figura 3.16A – Exemplos de “ períodos ” ou “ unidades translacionais ” formados pelos vetores mínimos da malha $\#A\{P\}$. Vale observar que as diagonais dos losangos são iguais aos lados dos retângulos.



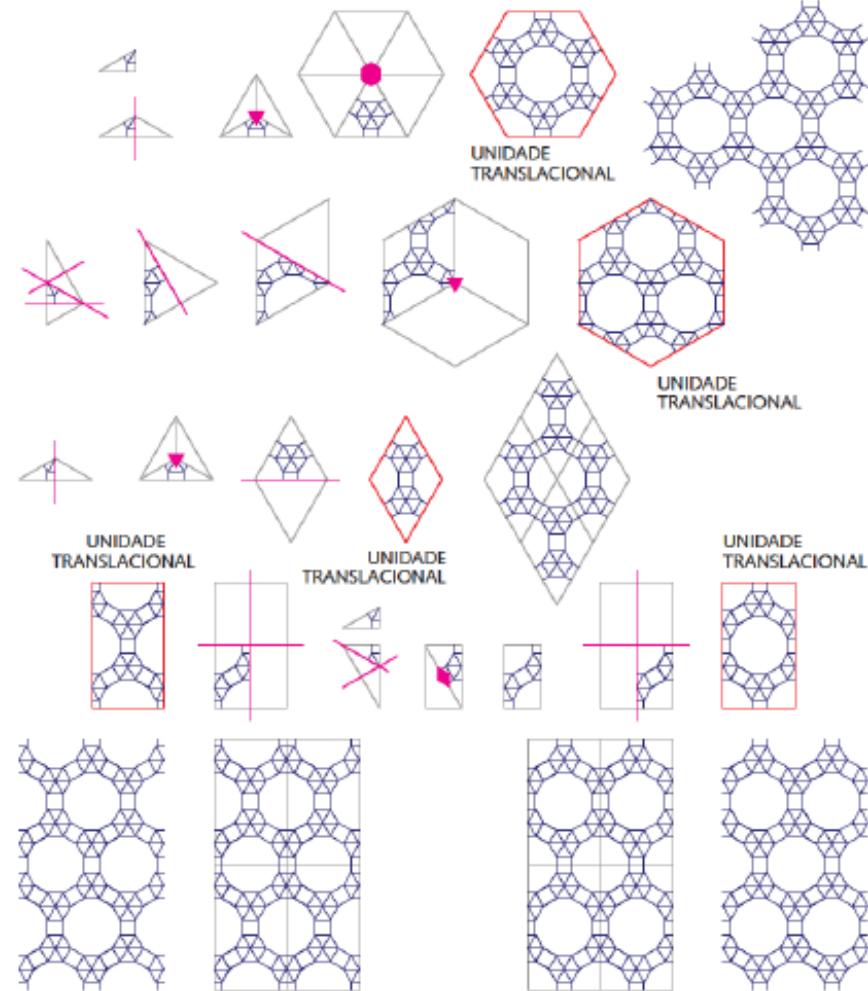
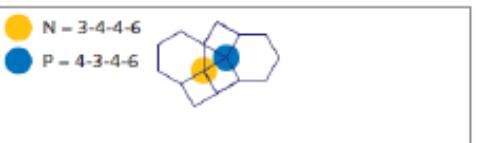
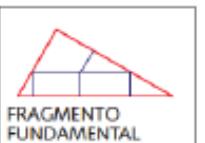


FRAGMENTO FUNDAMENTAL

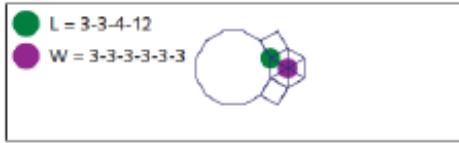




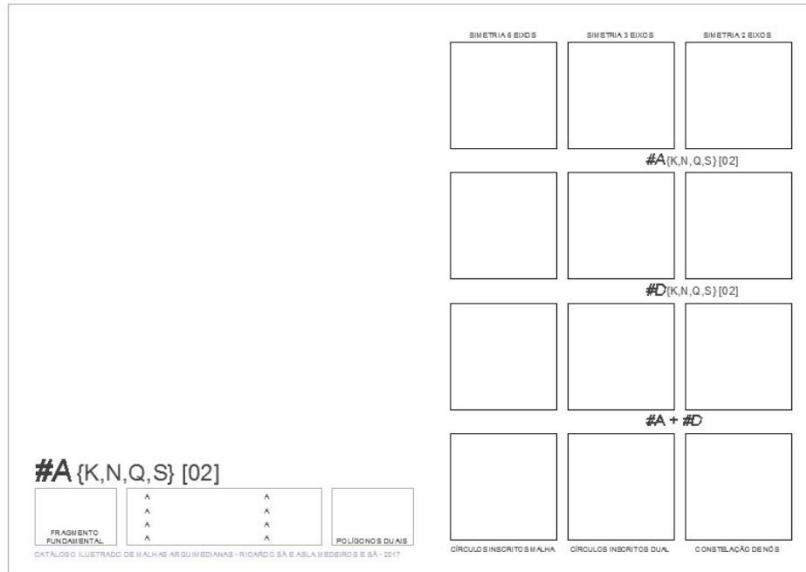
#A_{N,P} [01]



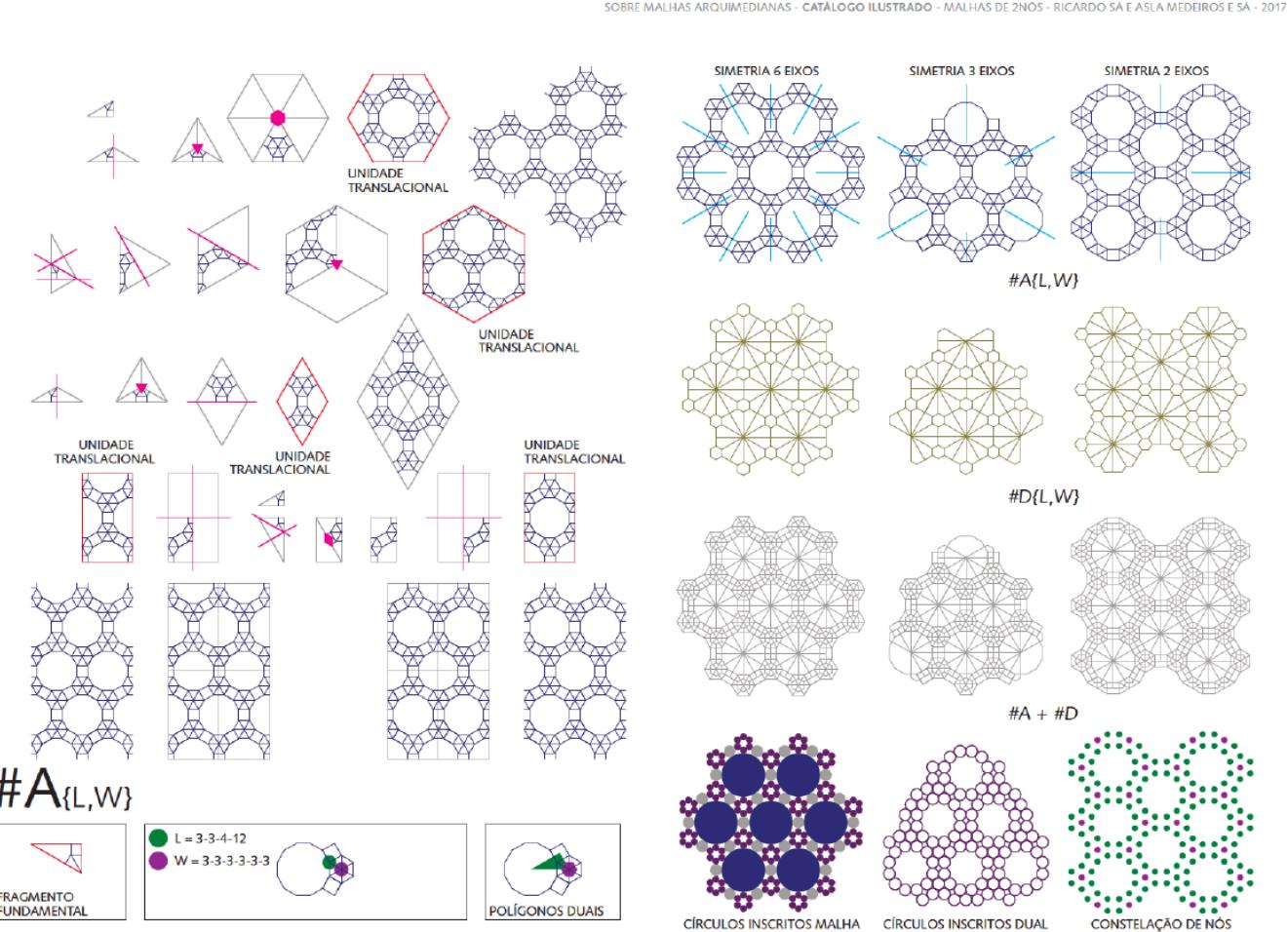
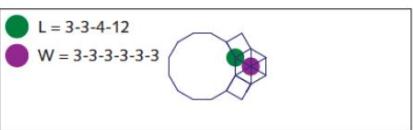
#A_{L,W}



O catálogo SOBRE MALHAS ARQUIMEDIANAS



#A_{L,W}

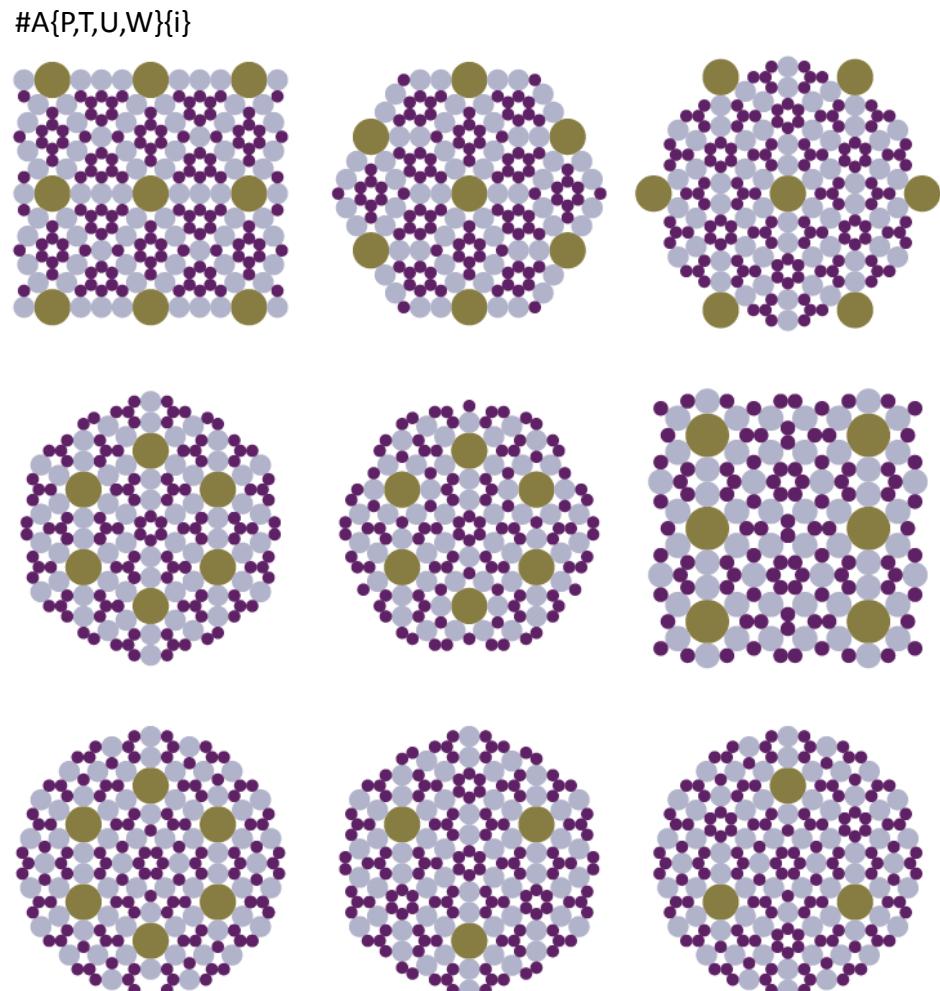
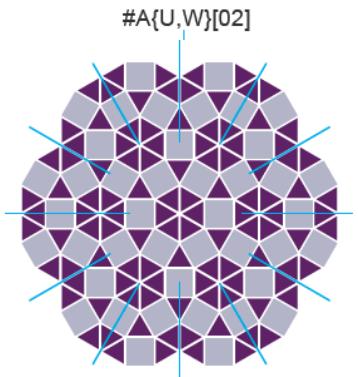
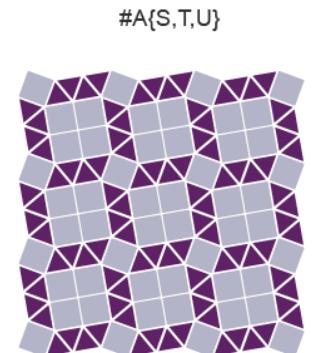
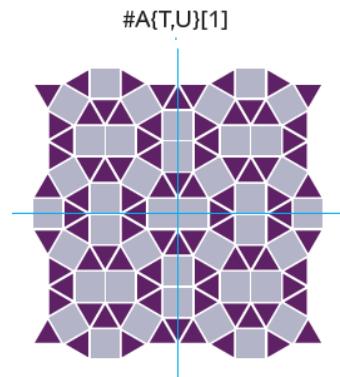
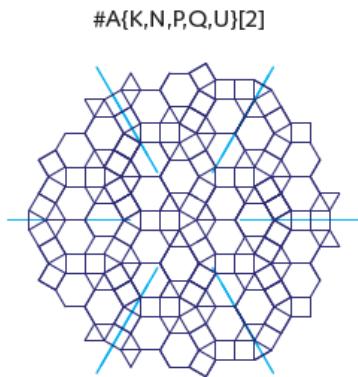
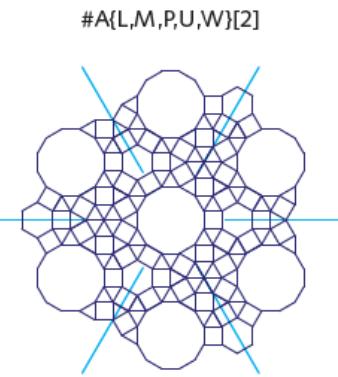
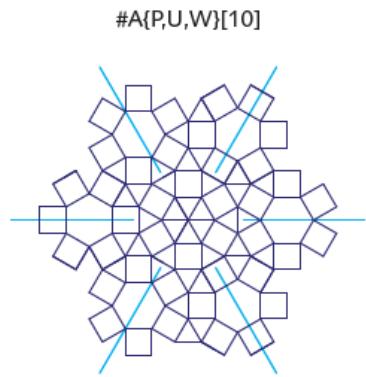


LISTA DE MALHAS DO CATÁLOGO ILUSTRADO

GRUPO 1	#A{QV}	#A{KQV}[3]	#A{PTU}[4]	#A{TUV}	#A{KQRV}[3]	#A{PTUW}[16]	SEIS NÓS
UM NÓ	#A{QW}	#A{KQW}[1]	#A{PTU}[5]	#A{TUW}[1]	#A{KQRV}[4]	#A{PTUW}[17]	#A{KNQSTU}
#A{G}	#A{RV}[1]	#A{KQW}[2]	#A{PTU}[6]	#A{TUW}[2]	#A{LMST}	#A{PTUW}[18]	#A{LMPTUW}
#A{H}	#A{RV}[2]	#A{LMU}	#A{PTU}[7]	#A{TUW}[3]	#A{LMUW}	#A{QRVW}	#A{LNPRST}
#A{J}	#A{ST}	#A{LPT}	#A{PTW}	#A{TUW}[4]	#A{LTUW}	#A{STUW}[1]	#A{NPQTUV}
#A{K}	#A{TU}[1]	#A{LTW}	#A{PUW}[1]	#A{TUW}[5]	#A{MPUW}	#A{STUW}[2]	#A{NPQTUW}
#A{P}	#A{TU}[2]	#A{LUW}[1]	#A{PUW}[2]	#A{TUW}[6]	#A{NPQU}	#A{STUW}[3]	#A{NPRSTU}
#A{R}	#A{TU}[3]	#A{LUW}[2]	#A{PUW}[3]	#A{TUW}[7]	#A{NPTU}[1]		#A{NPRTUV}
#A{S}	#A{TW}	#A{MPU}	#A{PUW}[4]	#A{TUW}[8]	#A{NPTU}[2]	GRUPO 4	#A{NQTVW}
#A{T}	#A{UW}[1]	#A{MUW}	#A{PUW}[5]	#A{TUW}[9]	#A{NPUW}[1]	CINCO NÓS	
#A{U}	#A{UW}[2]	#A{NPR}[1]	#A{PUW}[6]	#A{TUW}[10]	#A{NPUW}[2]	#A{GHLPU}	SETE NÓS
#A{V}	#A{UW}[3]	#A{NPR}[2]	#A{PUW}[7]	#A{TUW}[11]	#A{NQRS}[1]	#A{KNPQU}[1]	#A{HLMNQ TU}
#A{W}	#A{UW}[4]	#A{NPR}[3]	#A{PUW}[8]	#A{TUW}[12]	#A{NQRS}[2]	#A{KNPQU}[2]	#A{NPRSTUW}[1]
	#A{UW}[5]	#A{NPS}[1]	#A{PUW}[9]	#A{TUW}[13]	#A{NQSU}	#A{KNQSU}	#A{NPRSTUW}[2]
DOIS NÓS	#A{VW}[1]	#A{NPS}[2]	#A{PUW}[10]	#A{TUW}[14]	#A{NRTW}	#A{LMPTU}[1]	
#A{GM}[1]	#A{VW}[2]	#A{NPS}[3]	#A{PUW}[11]	#A{TUW}[15]	#A{NTVW}	#A{LMPTU}[2]	
#A{GM}[2]	#A{VW}[3]	#A{NPU}[1]	#A{PUW}[12]		#A{PTUW}[1]	#A{LMPUW}[1]	
#A{HP}[1]		#A{NPU}[2]	#A{PUW}[13]	GRUPO 3	#A{PTUW}[2]	#A{LMPUW}[2]	
#A{HP}[2]	GRUPO 2	#A{NQR}[1]	#A{PUW}[14]	QUATRO NÓS	#A{PTUW}[3]	#A{LMTUW}[1]	
#A{LW}	TRES NÓS	#A{NQR}[2]	#A{PUW}[15]	#A{GLMT}[1]	#A{PTUW}[4]	#A{LMTUW}[2]	
#A{NP}[1]	#A{GLM}	#A{NQT}	#A{QRV}[1]	#A{GLMT}[2]	#A{PTUW}[5]	#A{LMTUW}[3]	
#A{NP}[2]	#A{GLP}	#A{NQU}	#A{QRV}[2]	#A{GLUW}	#A{PTUW}[6]	#A{LPTUW}	
#A{NP}[3]	#A{GMP}	#A{NRS}[1]	#A{QRV}[3]	#A{GMUW}	#A{PTUW}[7]	#A{LSTUW}	
#A{NR}[1]	#A{HLW}	#A{NRS}[2]	#A{QVW}[1]	#A{HLMP}	#A{PTUW}[8]	#A{NPQTU}[1]	
#A{NR}[2]	#A{HNR}	#A{NTV}[1]	#A{QVW}[2]	#A{HNQT}	#A{PTUW}[9]	#A{NPQTU}[2]	
#A{PT}	#A{KQR}[1]	#A{NTV}[2]	#A{QVW}[3]	#A{HNQU}	#A{PTUW}[10]	#A{NPRST}	
#A{PU}[1]	#A{KQR}[2]	#A{NUV}]	#A{RVW}[1]	#A{KNQT}	#A{PTUW}[11]	#A{NPTUW}	
#A{PU}[2]	#A{KQR}[3]	#A{NUW}	#A{RVW}[2]	#A{KNQU}[1]	#A{PTUW}[12]	#A{NQTVW}	
#A{PU}[3]	#A{KQR}[4]	#A{PTU}[1]	#A{RVW}[3]	#A{KNQU}[2]	#A{PTUW}[13]	#A{NRSTW}	
#A{PU}[4]	#A{KQV}[1]	#A{PTU}[2]	#A{STU}	#A{KQRV}[1]	#A{PTUW}[14]	#A{NSTVW}	
#A{QR}	#A{KQV}[2]	#A{PTU}[3]	#A{STW}	#A{KQRV}[2]	#A{PTUW}[15]	#A{PSTUW}	

Capítulo 4

FAMÍLIAS DE MALHAS



Sibdivisão e agrupamento de polígonos

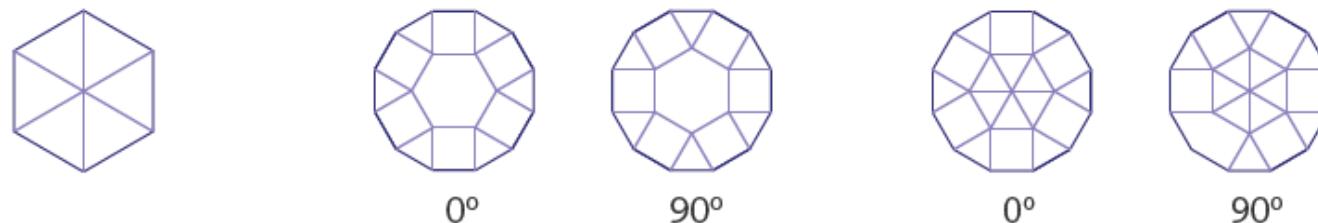


Figura 4.4 – Exemplo das possíveis subdivisões do hexágono e dodecágono.

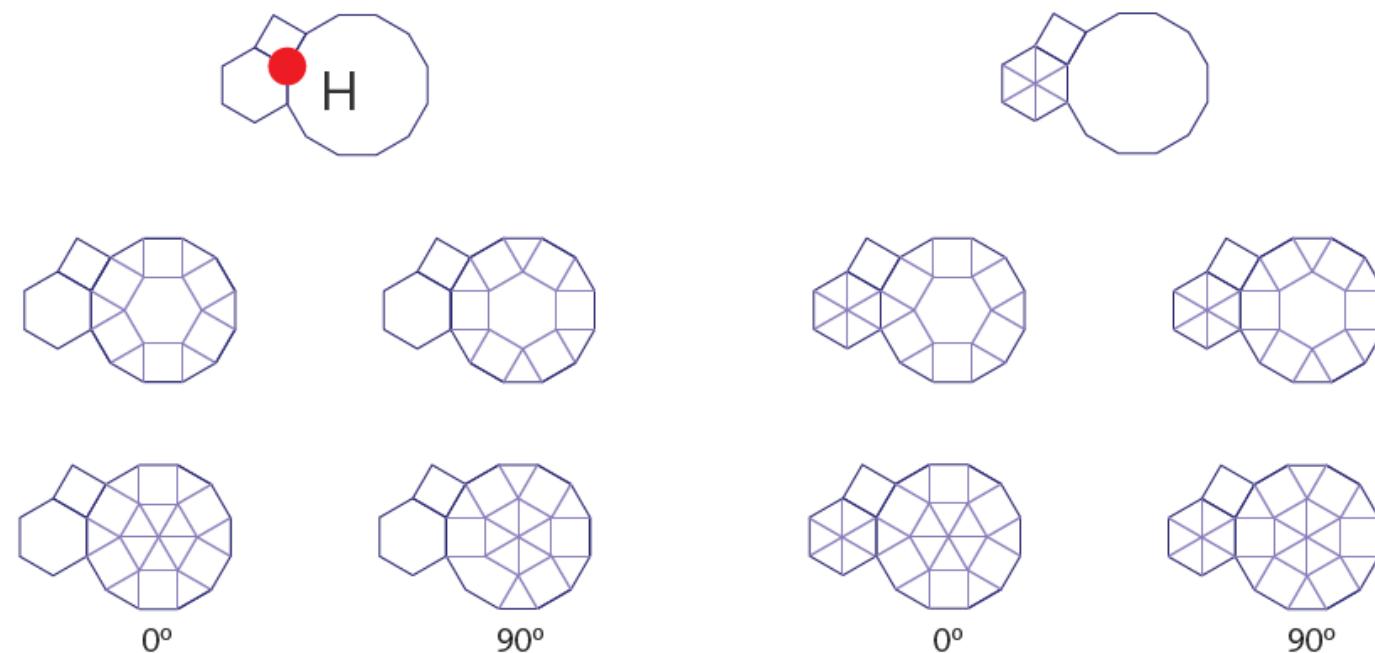


Figura 4.5 – Exemplo das possíveis subdivisões do nó H.

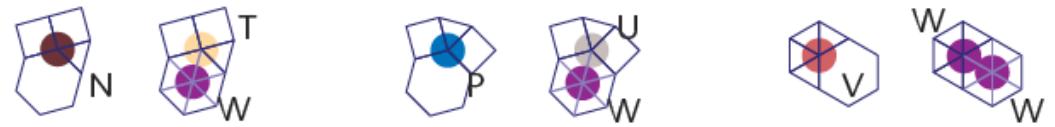


Figura 4.7 – Subdivisão dos nos N, P e V.



Figura 4.8 – Subdivisões possíveis dos nos K, Q e R.

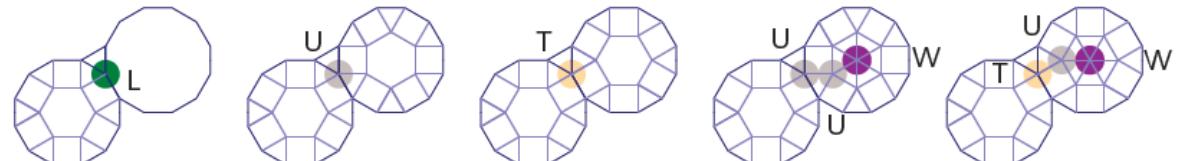


Figura 4.9 – Subdivisões possíveis dos nós L e M.

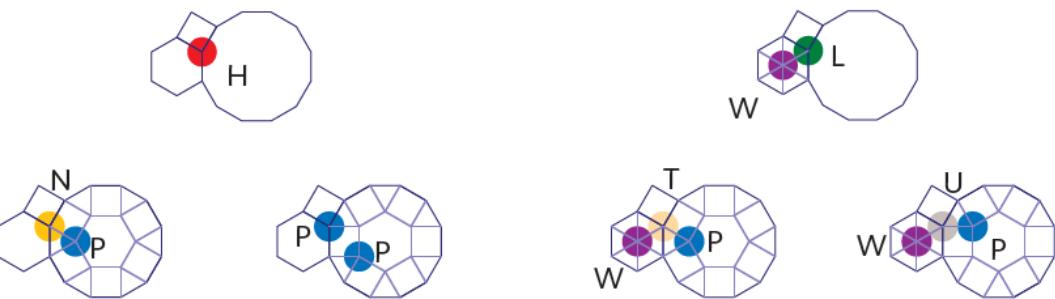


Figura 4.10 – As nove subdivisões do nó H.

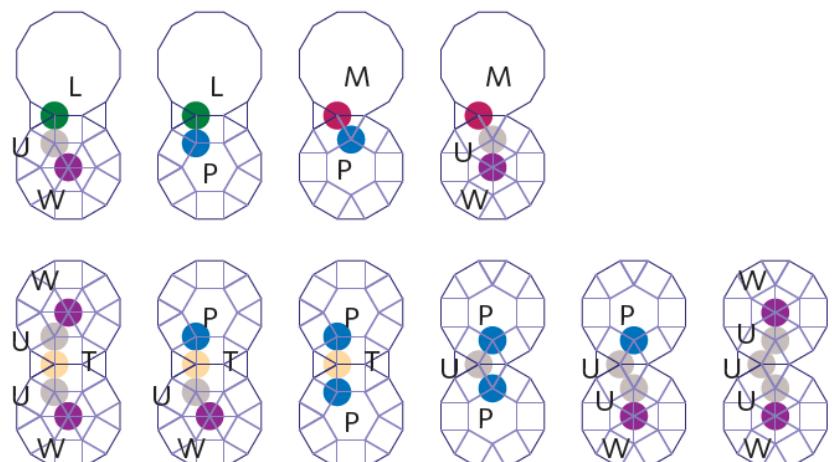
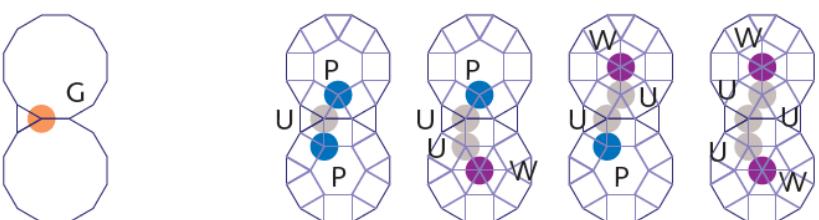


Figura 4.11 – As 14 possibilidades de subdivisão do nó G.

Famílias por subdivisão de polígonos

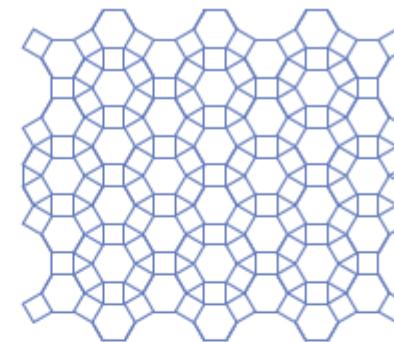
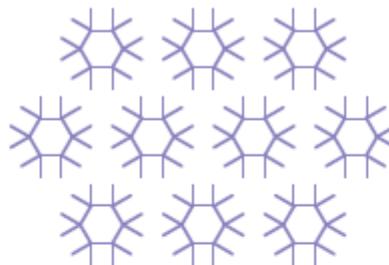
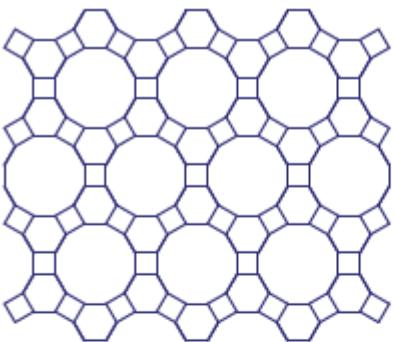


Figura 4.12A – A malha $\#A\{H\}$, ao subdividir seus dodecágono, complementando com as arestas ilustradas na figura do centro, se transforma na malha $\#A\{P\}$.

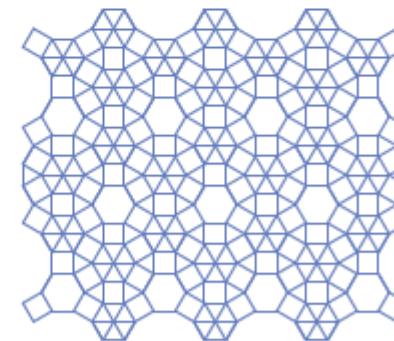
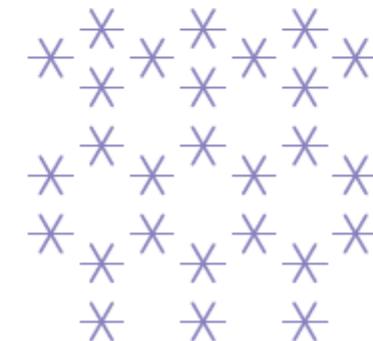
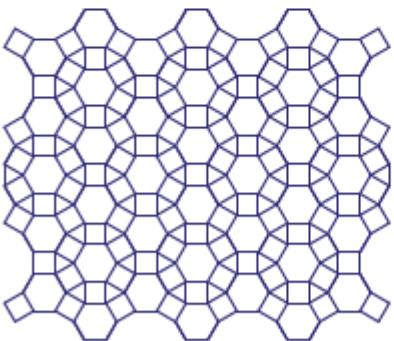
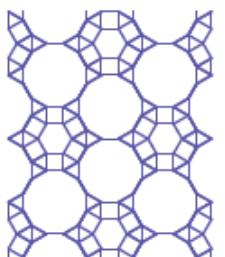
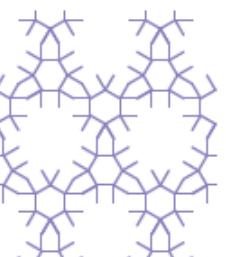


Figura 4.12B – A malha $\#A\{P\}$, quando divididos parte de seus hexágonos em triângulos resulta na malha $\#A\{P,U,W\}[2]$.



#A{G,L,P}



#A{P,T,U,W}[4]

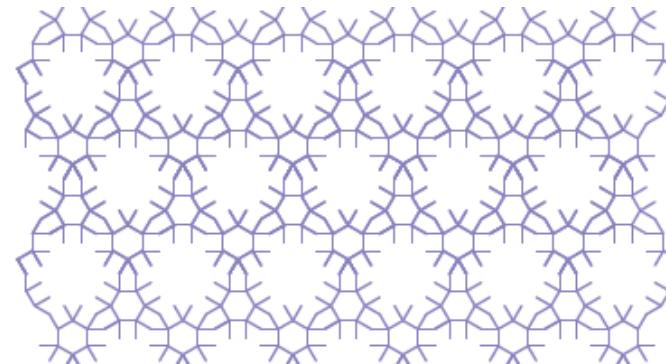
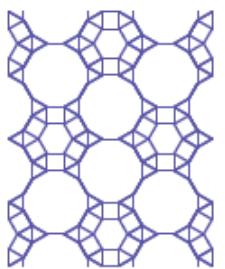
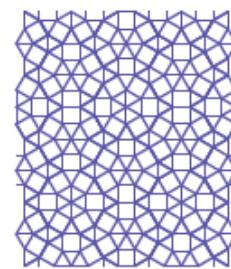
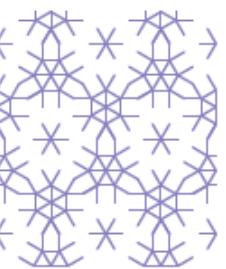


Figura 4.15B – Painel de arestas da subdivisão da malha #A{G,L,P} em direção à malha #A{P,T,U}[4].



#A{G,L,P}



#A{P,T,U,W}[9]

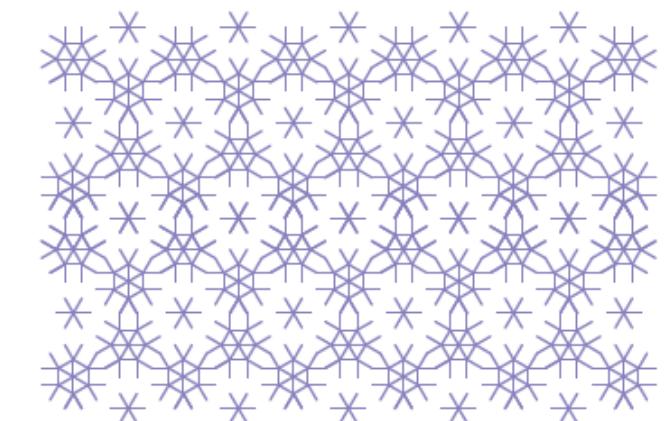
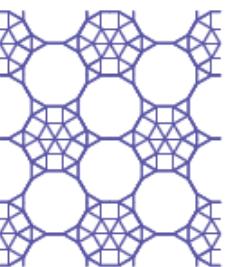
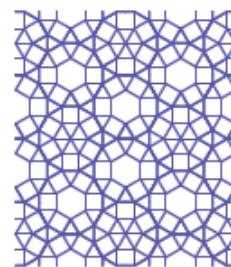
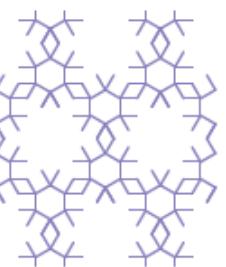


Figura 4.15C – Painel de arestas da subdivisão da malha #A{G,L,P} em direção à malha #A{ T,U,W}[9]



#A{G,L,U,W}



#A{P,U,W}[11]

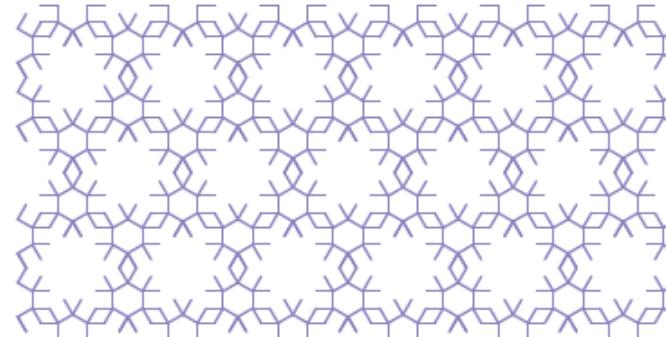
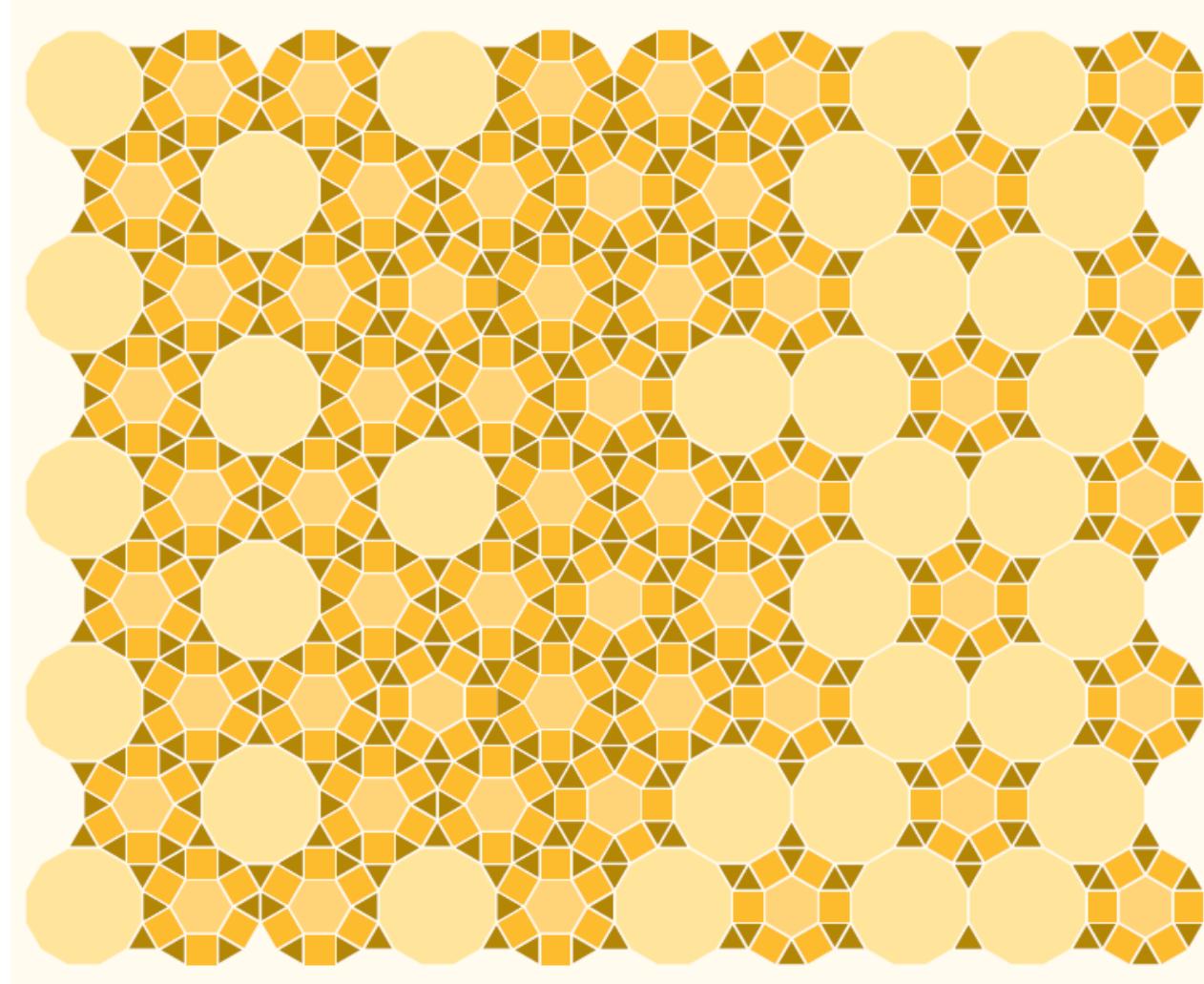
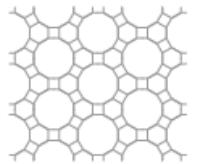


Figura 4.15D – Painel de arestas da subdivisão da malha #A{G,L,U,W})em direção à malha #A{P,T,W}[11]

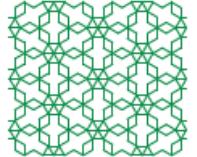
Capítulo 5

COMPOSIÇÃO E JUSTAPOSIÇÃO DE MALHAS

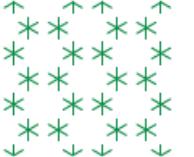
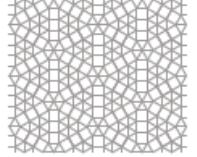




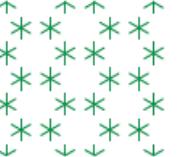
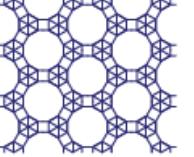
#A{H}



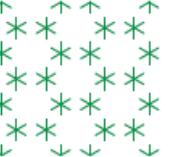
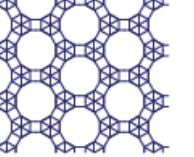
#A{T,U,W}[6]



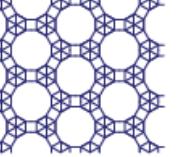
#A{L,W}



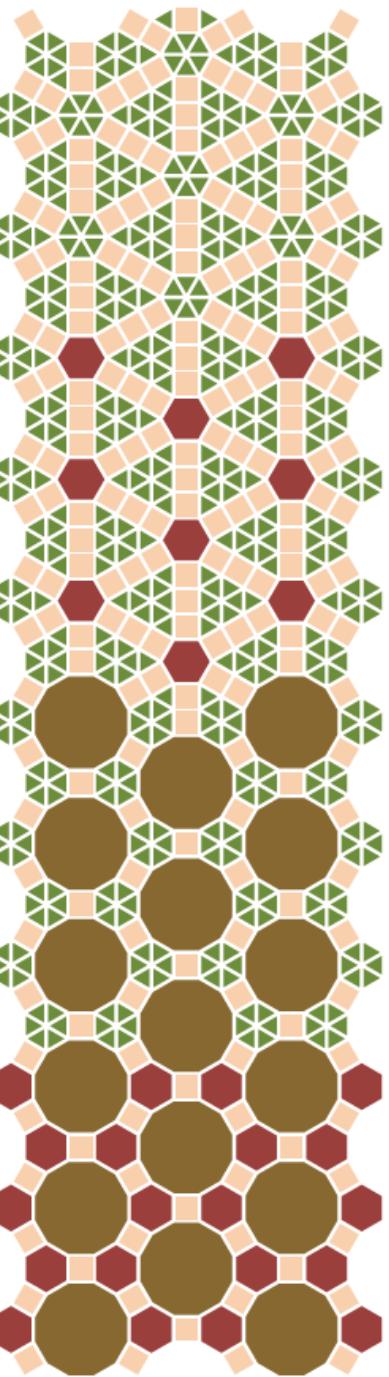
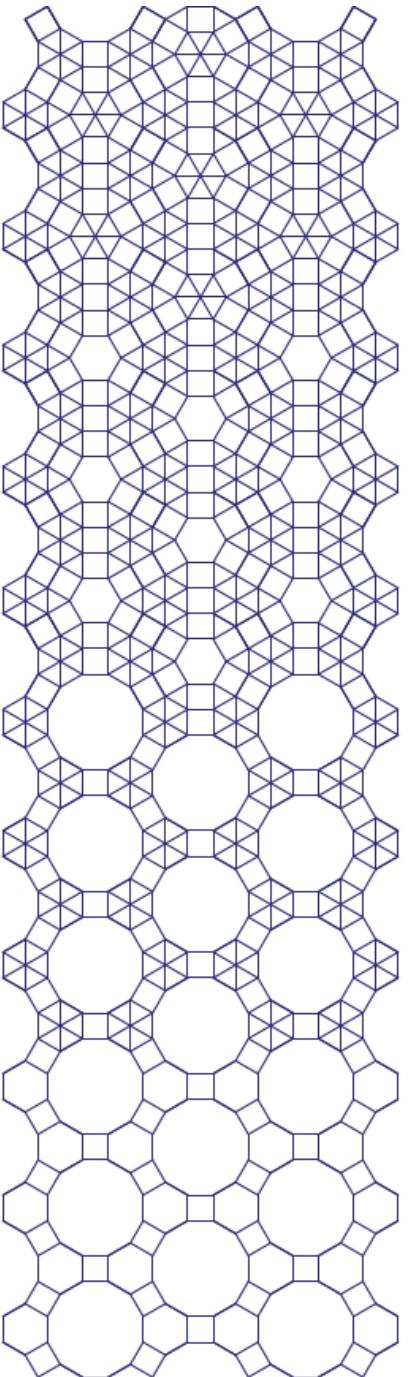
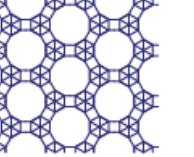
#A{L,W}



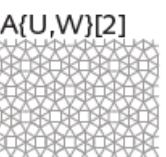
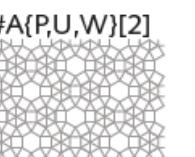
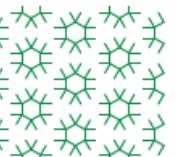
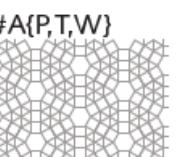
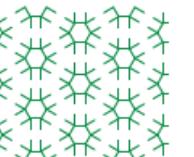
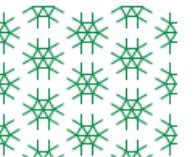
#A{L,W}



#A{L,W}



#A{T,U,W}[6]



#A{P,T,W}

#A{L,W}

#A{H}

Malhas multifamílias

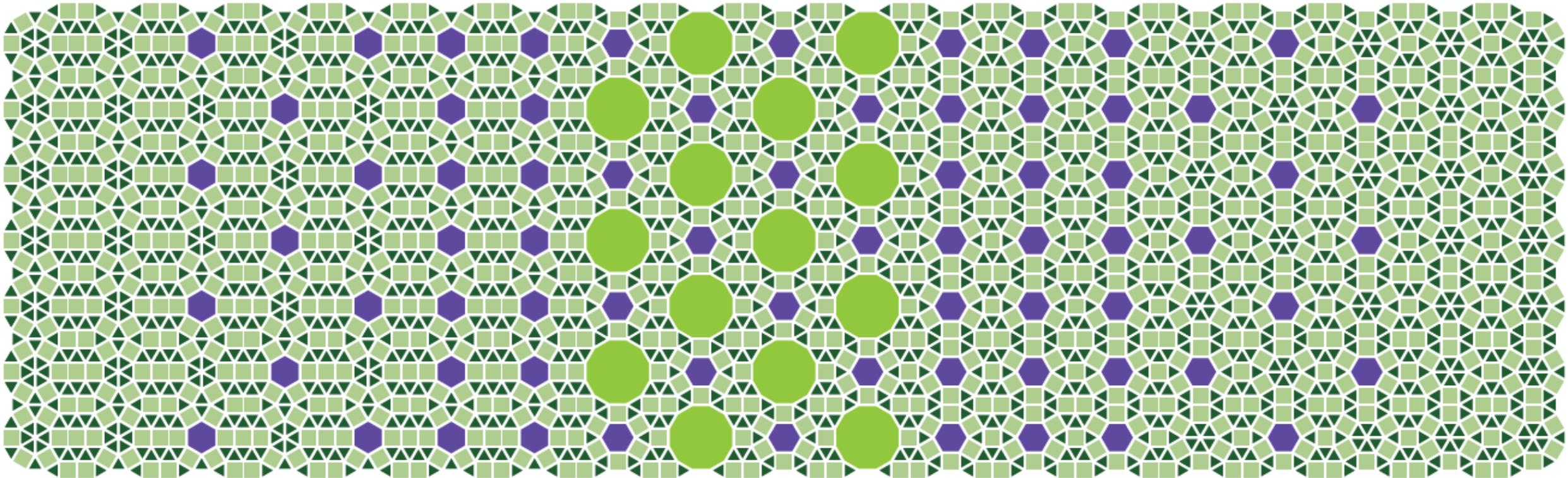
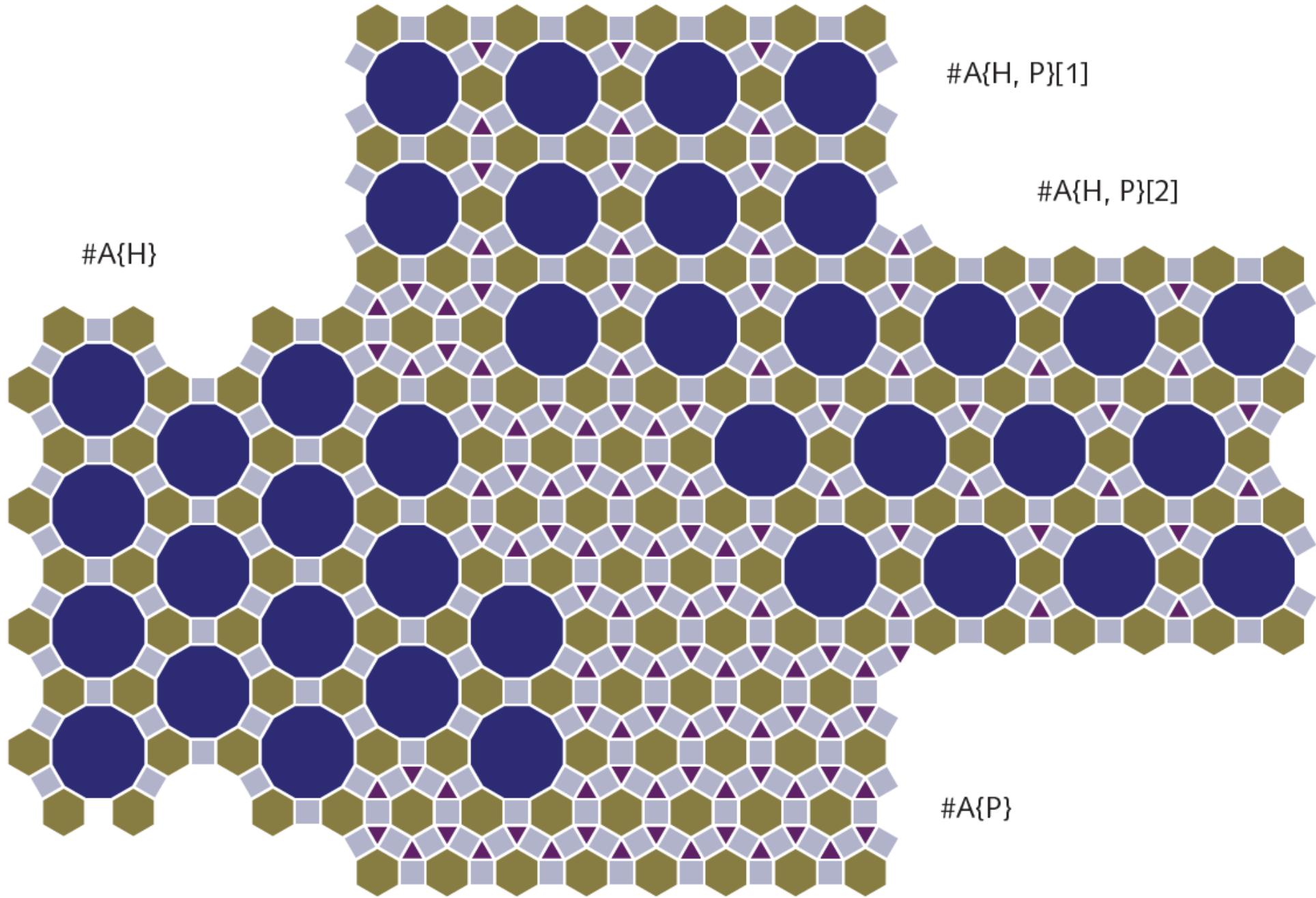
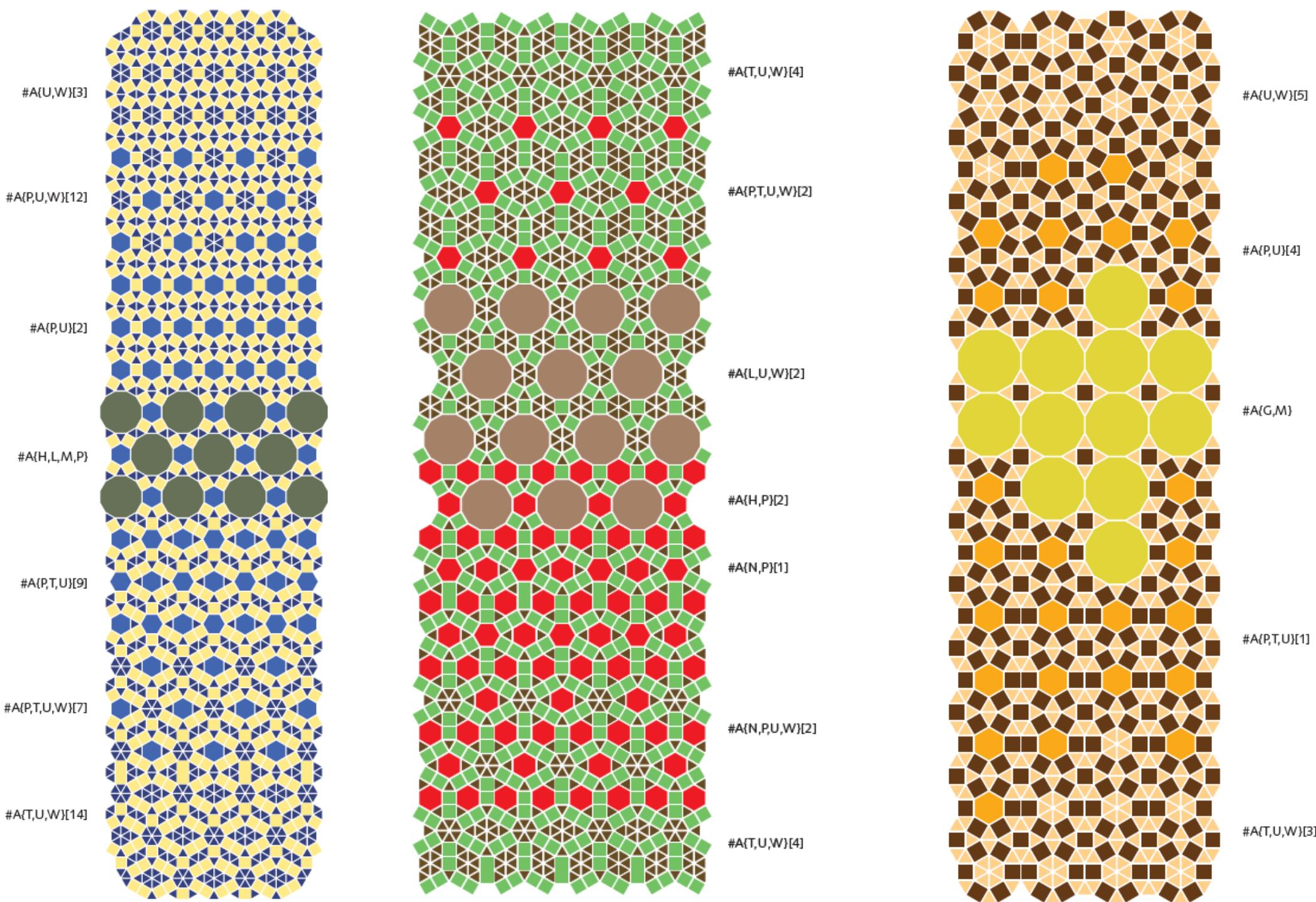
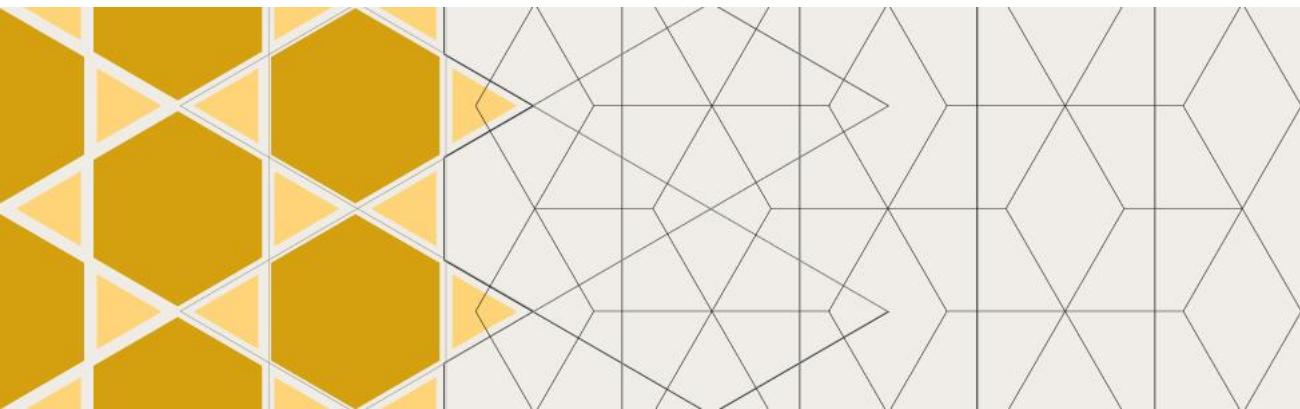


Figura 5.11A – Mosaico formado partindo da malha $\#A\{L,M,P,T,U\}$, dividida para a direita em um ramo e para a esquerda em outro ramo. Um bom exercício será identificar quais as malhas presentes neste friso.

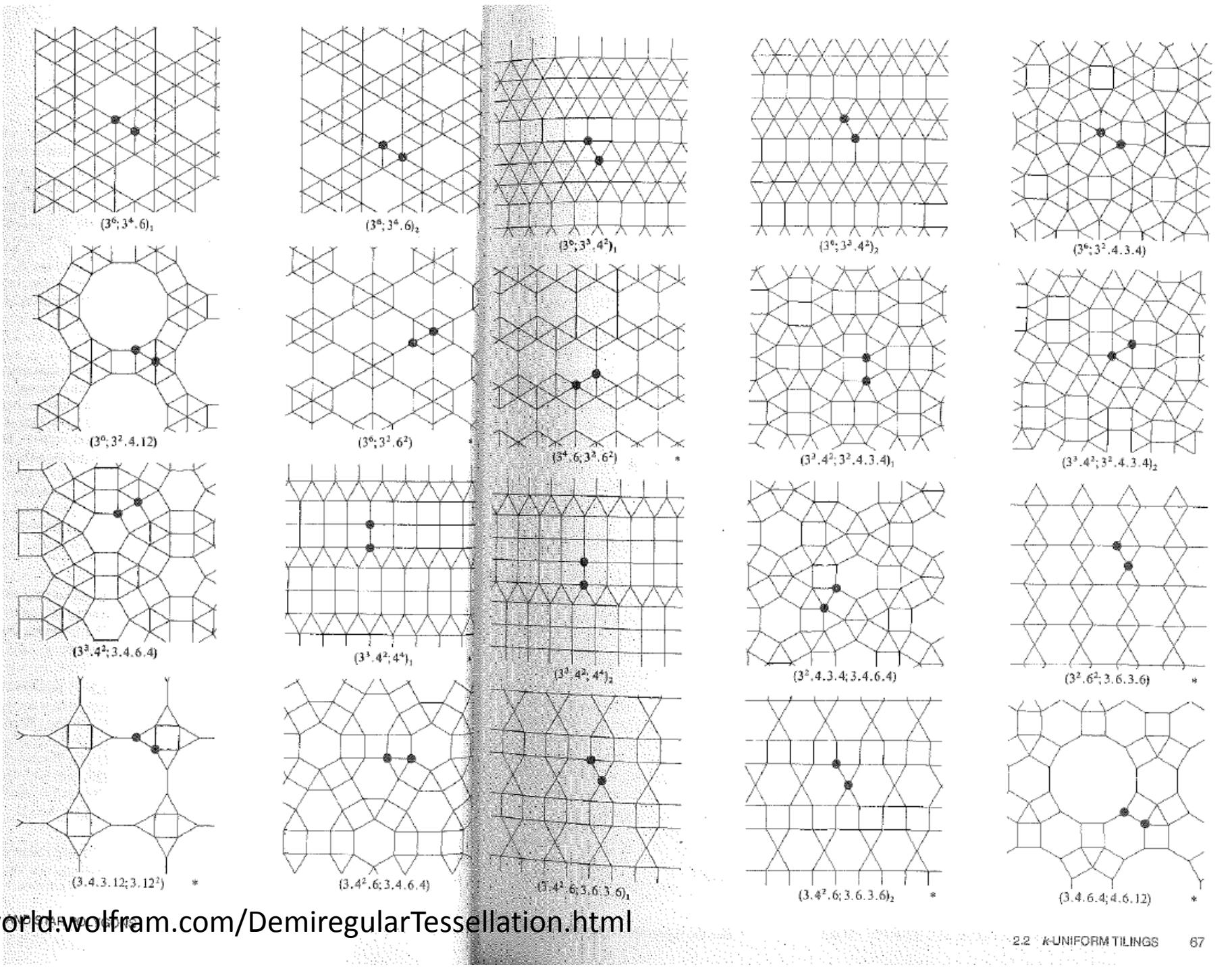






**SOBRE
MALHAS
ARQUIMEDIANAS**





Islamic Star Patterns in Absolute Geometry

CRAIG S. KAPLAN

University of Waterloo

and

DAVID H. SALESIN

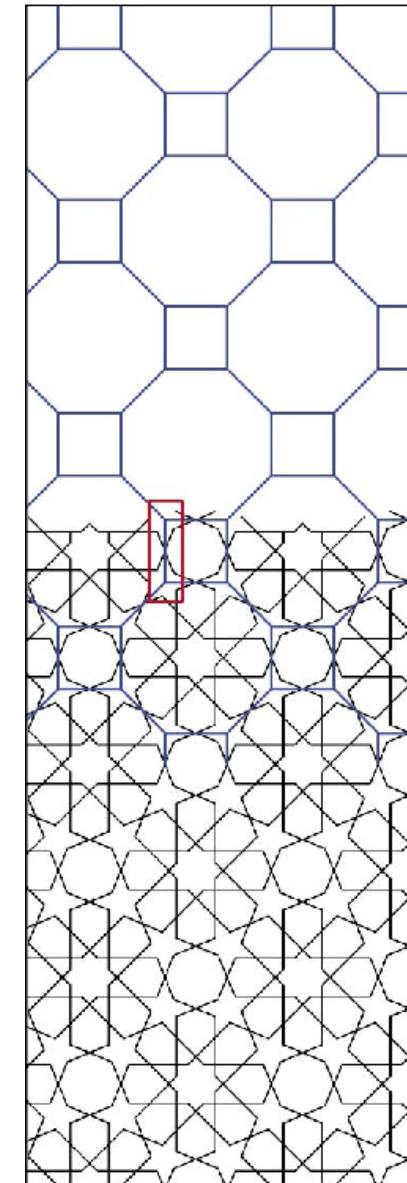
University of Washington and Microsoft Corporation

We present *Najm*, a set of tools built on the axioms of absolute geometry for exploring the design space of Islamic star patterns. Our approach makes use of a novel family of tilings, called “inflation tilings,” which are particularly well suited as guides for creating star patterns. We describe a method for creating a parameterized set of motifs that can be used to fill the many regular polygons that comprise these tilings, as well as an algorithm to infer geometry for any irregular polygons that remain. Erasing the underlying tiling and joining together the inferred motifs produces the star patterns. By choice, *Najm* is built upon the subset of geometry that makes no assumption about the behavior of parallel lines. As a consequence, star patterns created by *Najm* can be designed equally well to fit the Euclidean plane, the hyperbolic plane, or the surface of a sphere.

Categories and Subject Descriptors: I.3.5 [Computational Geometry and Object Modeling]: Geometric Algorithms, Languages and Systems; I.3.8 [Computer Graphics]: Applications; J.5 [Arts and Humanities]: Fine arts

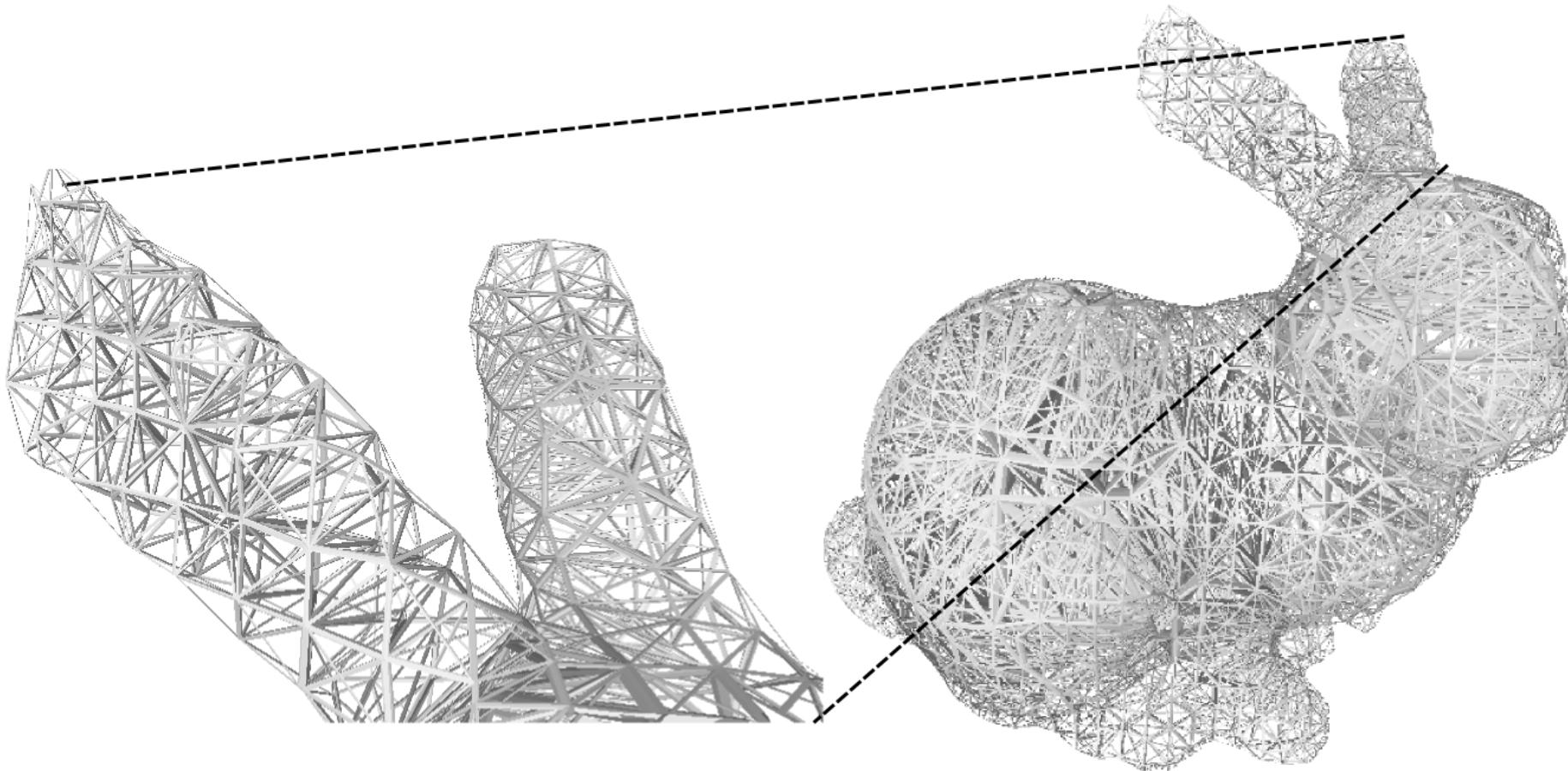
General Terms: Design, Algorithms

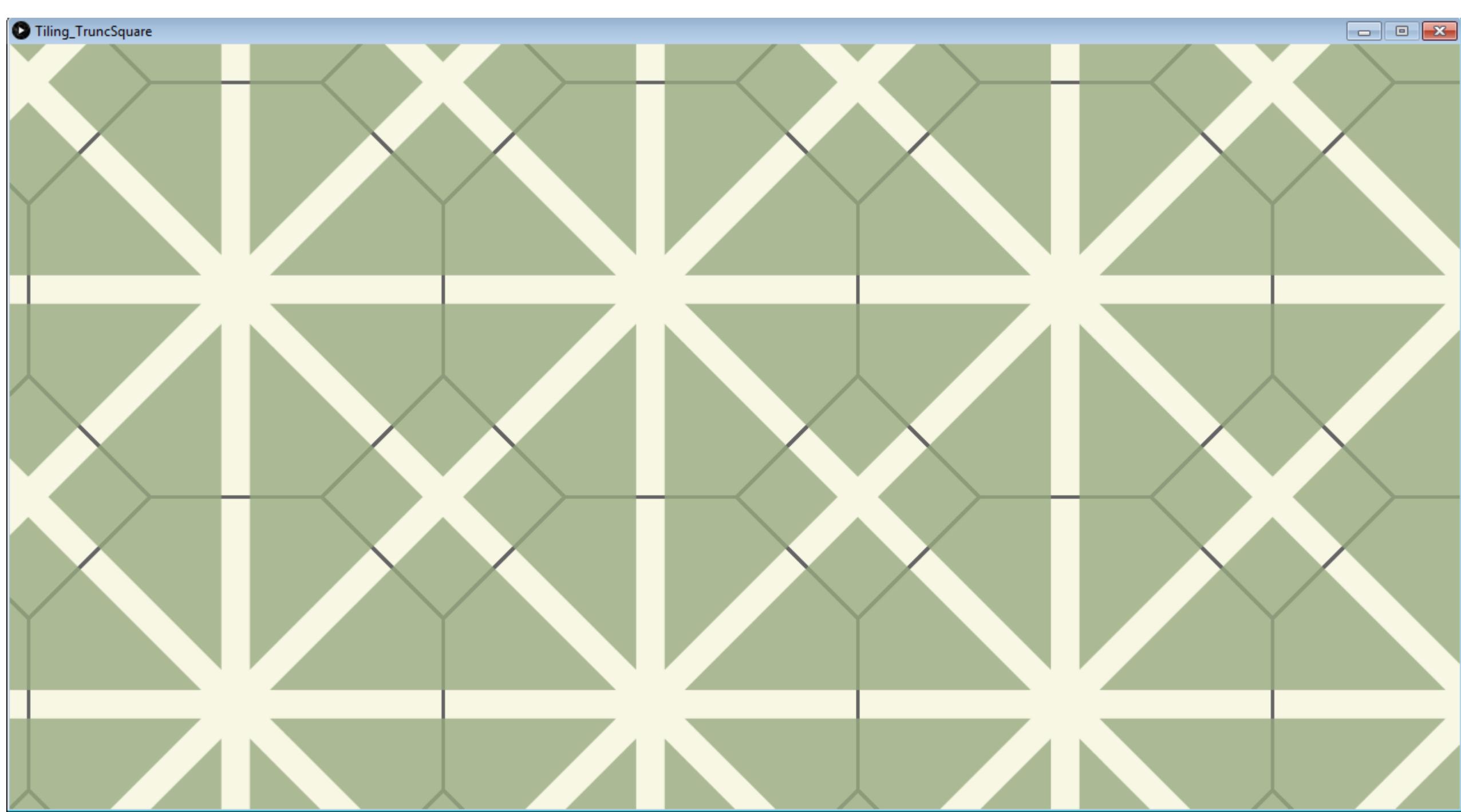
Additional Key Words and Phrases: Non-Euclidean geometry, symmetry, tessellations, tilings



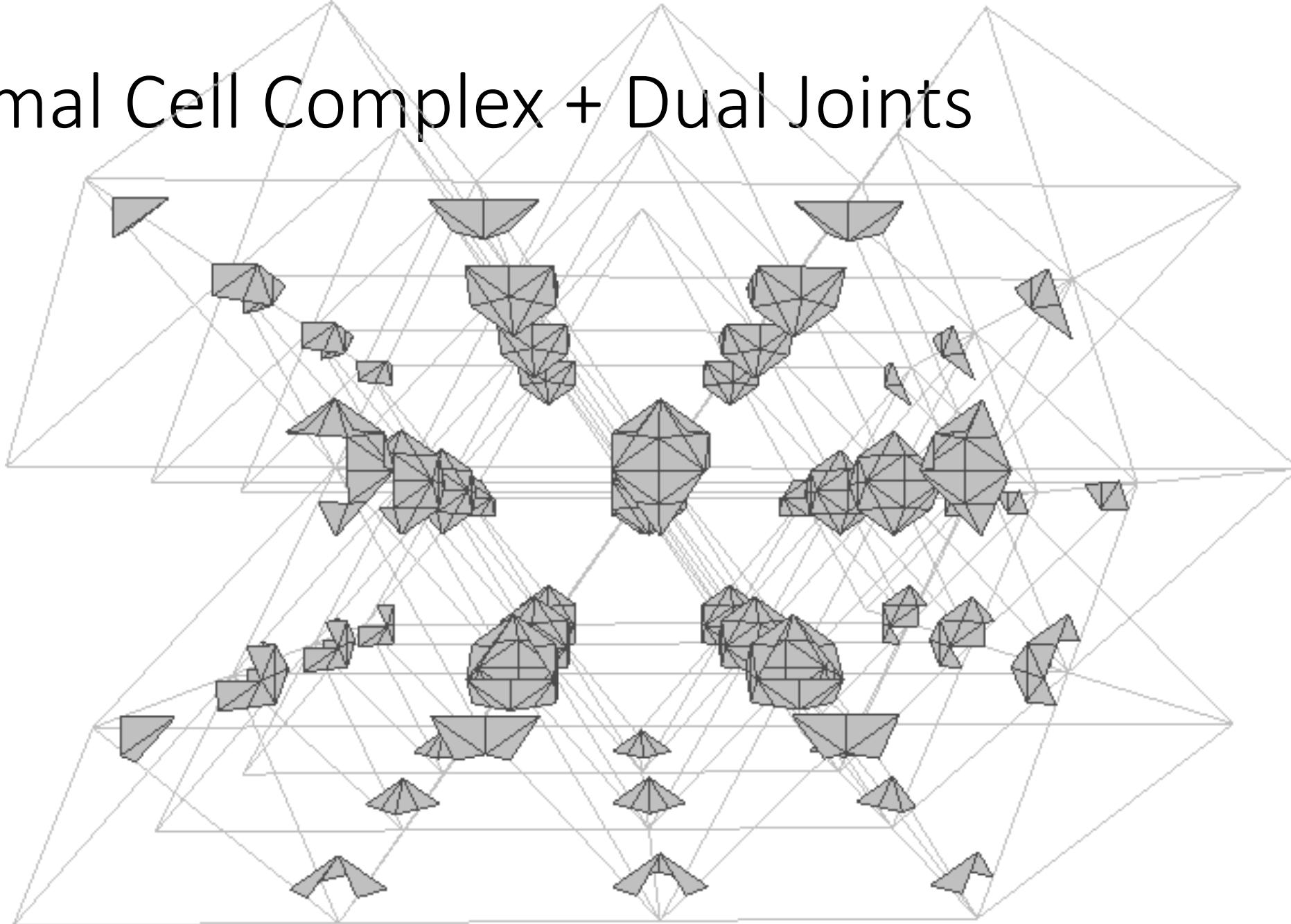
<http://www.cgl.uwaterloo.ca/csk/projects/starpatterns/>

Estruturas 3D

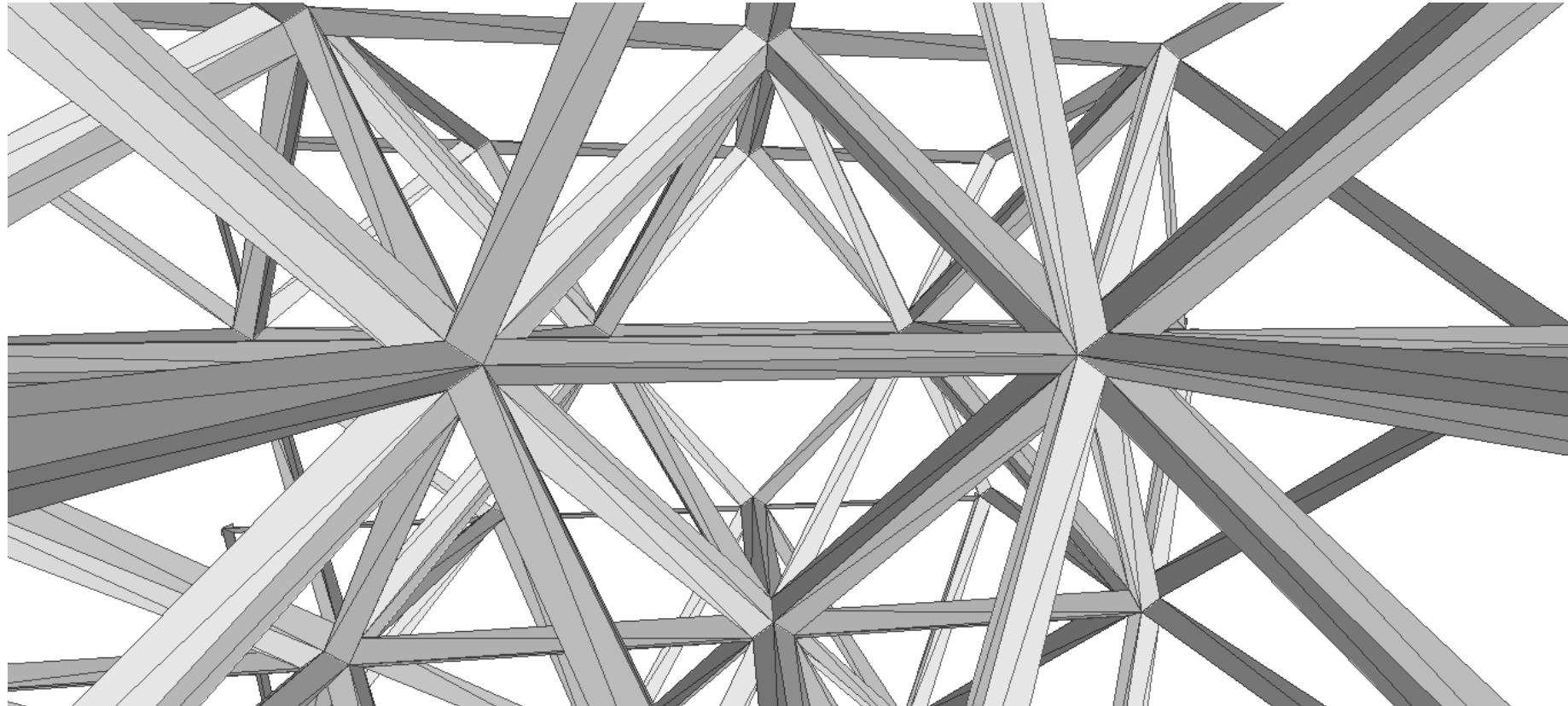




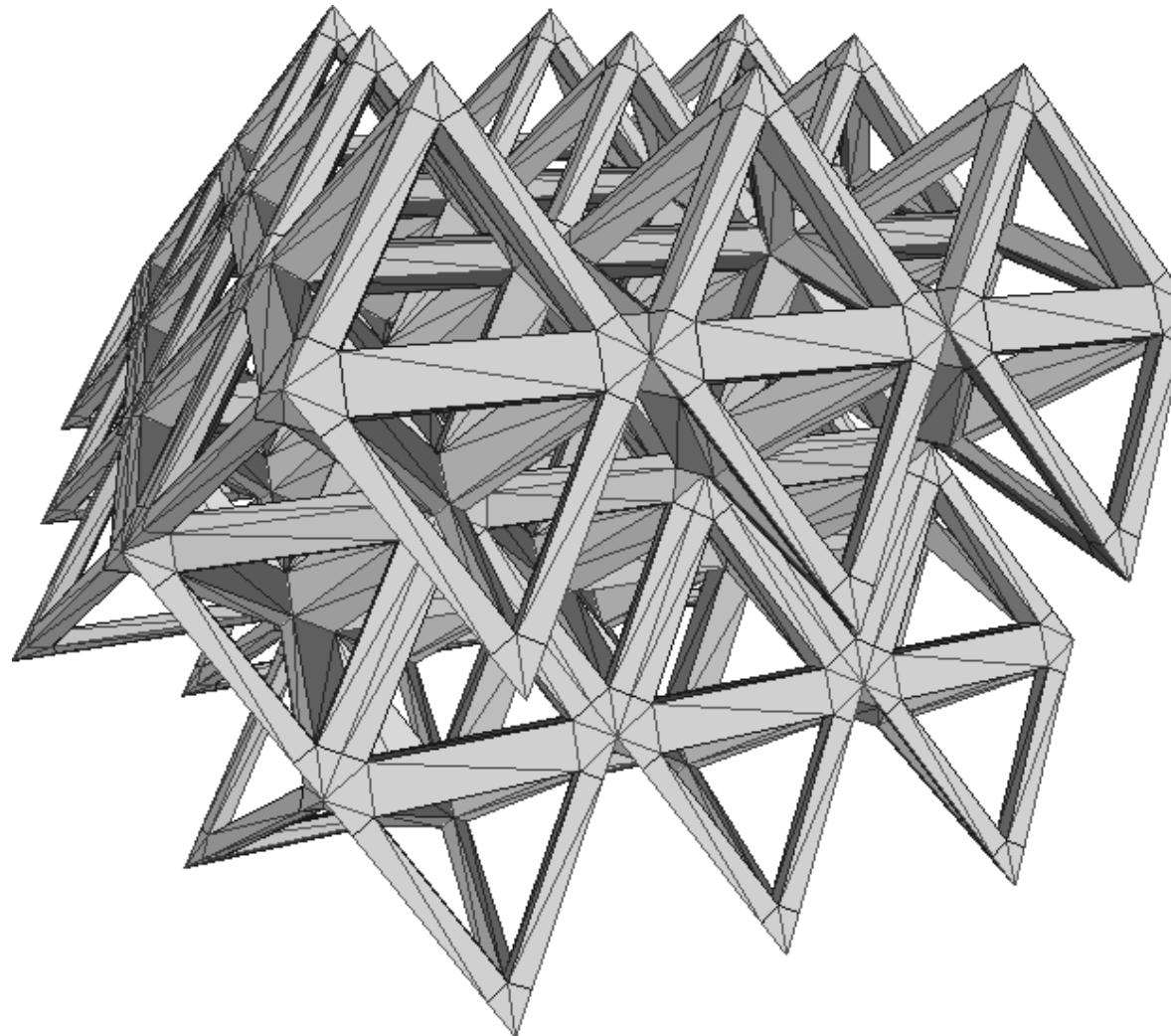
Primal Cell Complex + Dual Joints



Connections between adjacent Joints



Treating exceptions at the boundaries

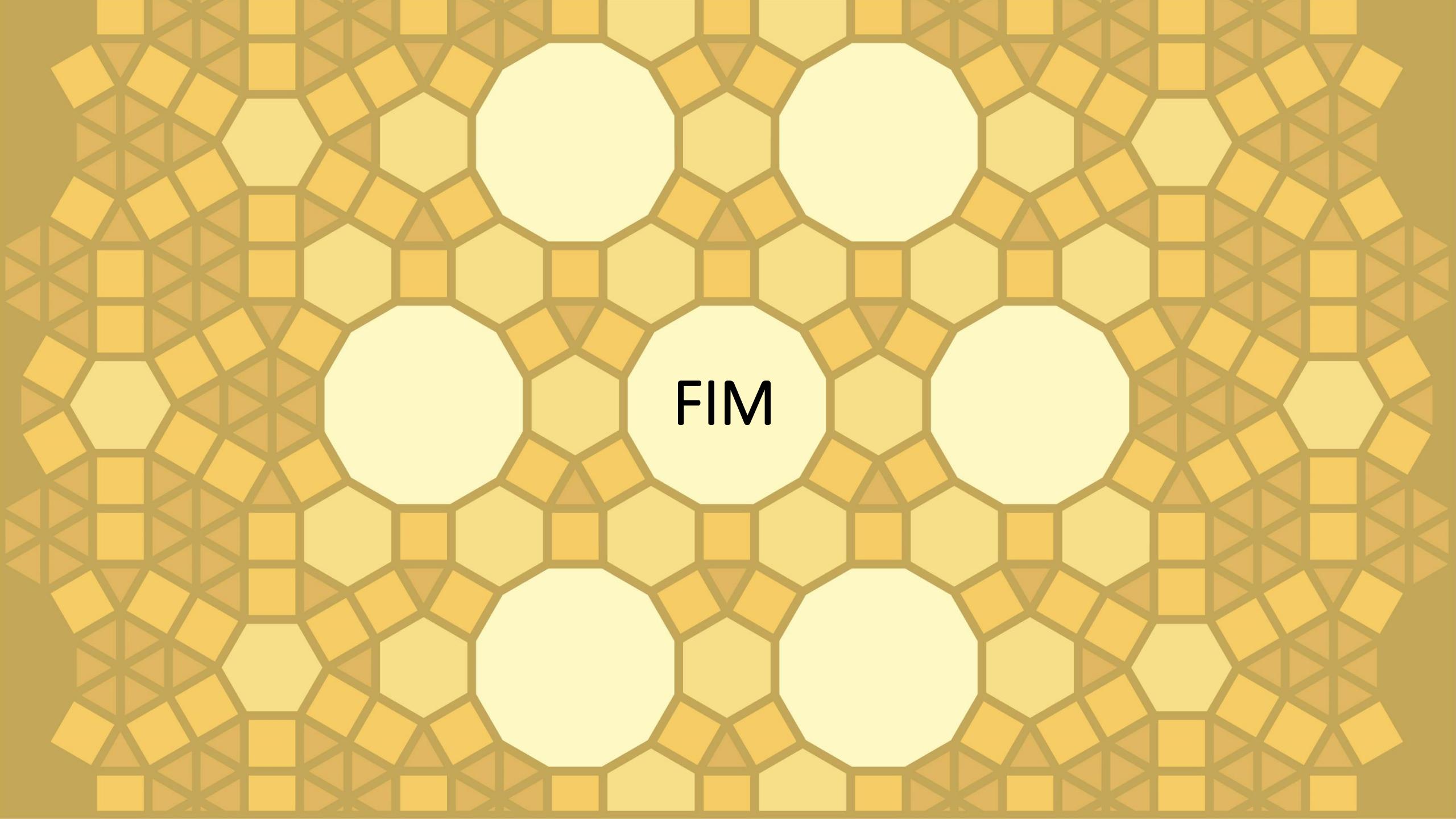


Referências

- [1] Keith Critchlow, “*Order in Space*”, Viking Press, New York, NY, 1969.
- [2] Ricardo Sa, “*Edros*”, Editora Projeto, 1982.
- [3] Branko Grunbaum e G.C. Shephard, “*Tilings and Patterns*”, W.H. Freeman & Co, New York, NY, 1987.
- [4] David Singer, “*Geometry: Plane and Fancy*”, Springer-Verlag, New York, 1998.
- [5] John H. Conway, Heidi Burgiel e Chaim Goodman-Strauss. “*The Symmetry of things*”, CRC Press, 2008.
- [6] Erika Brigitta Ledergerber-Ruoff, “*Isometrias e Ornamentos no Plano Euclidiano*”, Atual Editora, Editora da Universidade de São Paulo, 1982.
- [7] Craig S. Kaplan, “*Introductory Tiling Theory for Computer Graphics*”, Synthesis Lectures on Computer Graphics and Animation. 2009.

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