Scalable Load Balancing in Networked Systems

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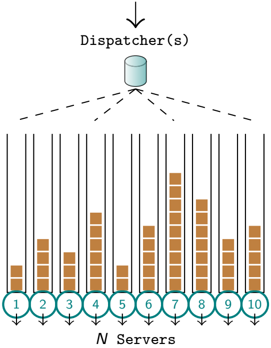
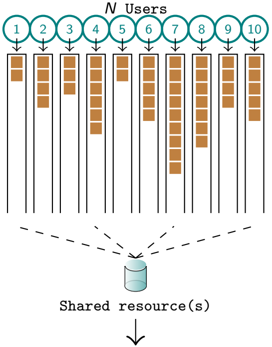
Based on joint work with Mark van der Boor, Johan van Leeuwaarden, Debankur Mukherjee & Phil Whiting

Two Fundamental Problems in Networked Systems

Centralized queue impractical/undesirable in large-scale networked systems

Achieve low delay and maintain low overhead as system grows large (with highly distributed queues)





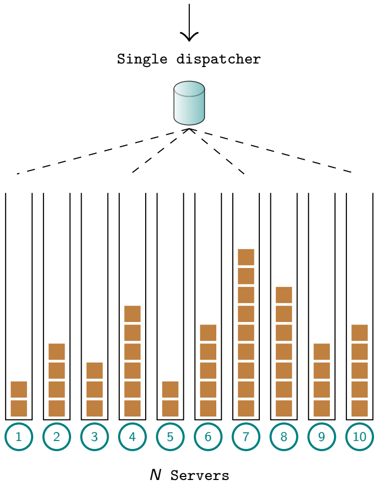
Load balancing/routing

Assign tasks from single arrival stream to be performed by one of several servers, each with their own queue

Resource allocation/scheduling

Allocate shared resource to execute tasks from one of several users, each with their own arrival stream and queue

Load Balancing/Routing in Parallel-Server Systems



Large-Scale Parallel-Server Systems: Some Examples

Supermarket checkout line

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Road toll plaza

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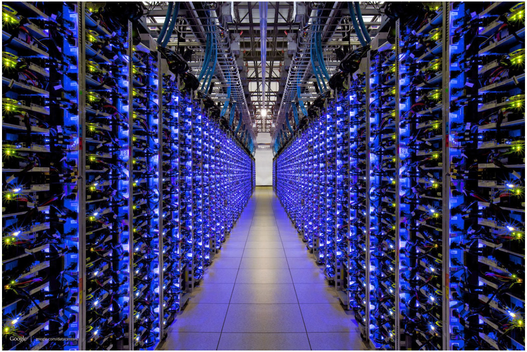

Data center

High-Level Outline

I. Scalability challenges and classical results
II. Asymptotic optimality and universality (no memory)
III. Reduction in communication overhead (memory)
IV. Heterogeneity issues and network scenarios

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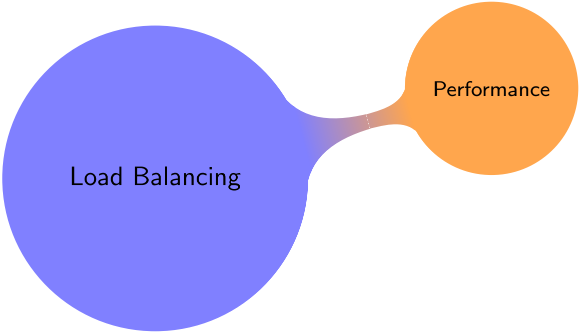

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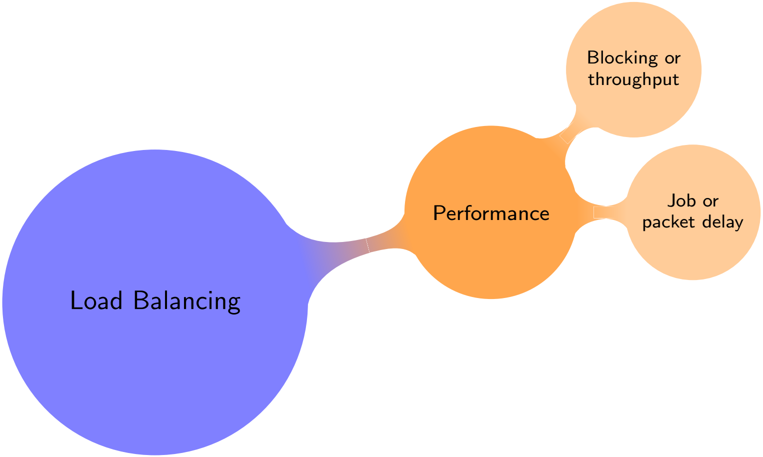

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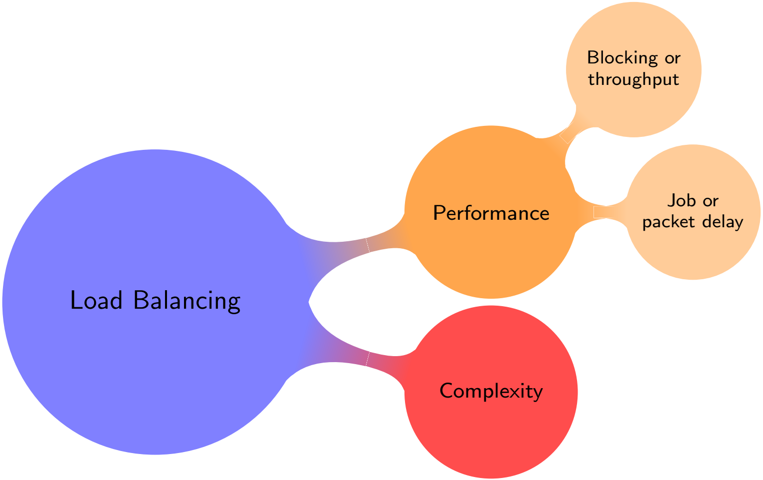

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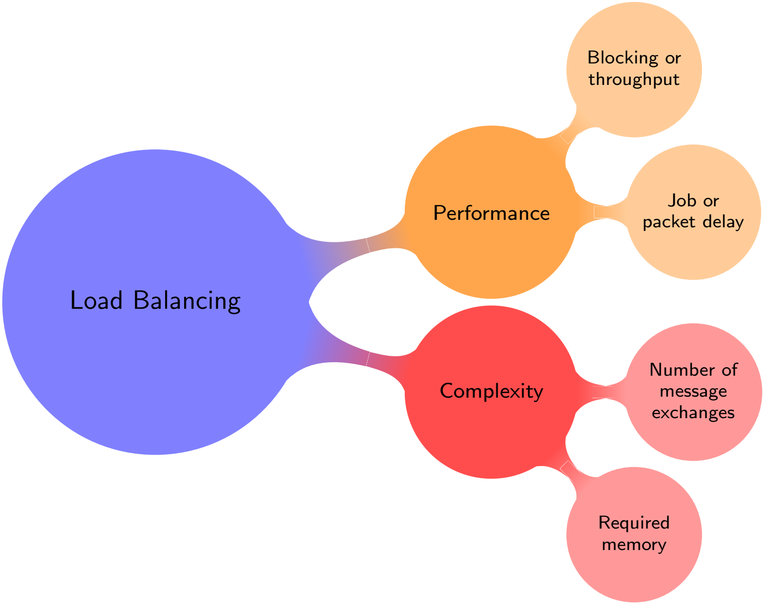
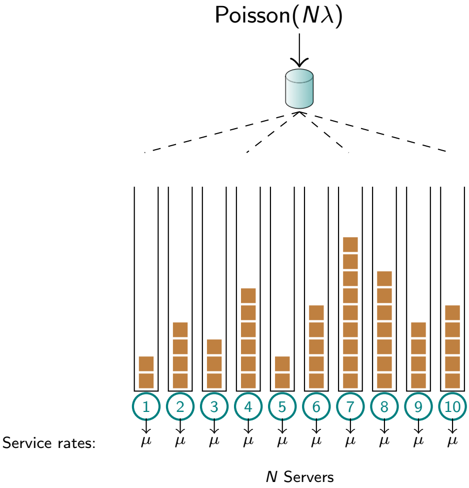


Purely Random Assignment



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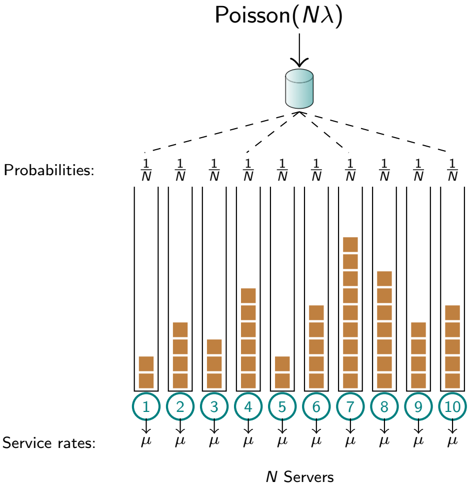

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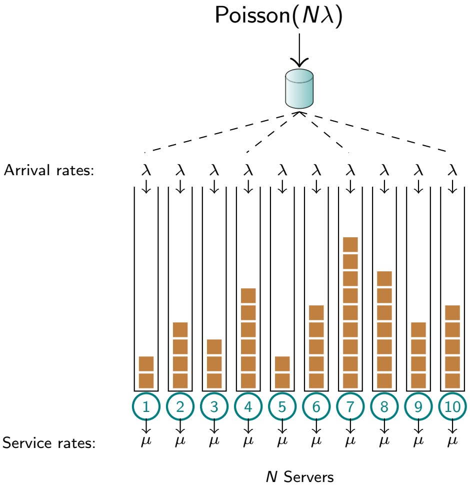

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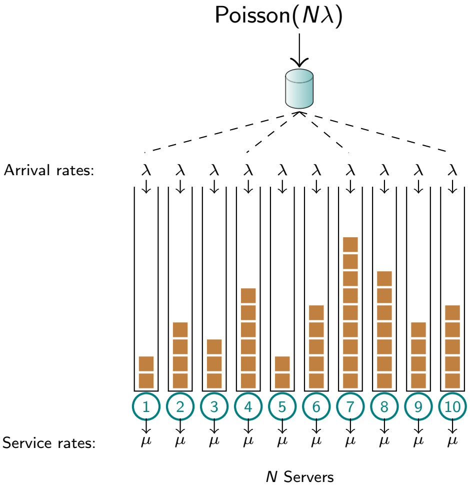
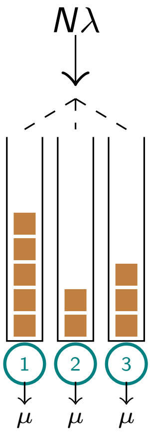


N independent M/M/1 queues with arrival rate λ and service rate μ

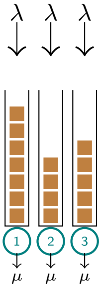
Load Balancing Scenarios



Separate Queues

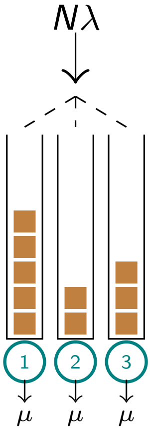
Strictly Random Routing

N $\times$ M/M/1



queue length $N \times \frac{\lambda^2}{\mu(\mu-\lambda)}$

waiting time $\frac{\lambda}{\mu(\mu-\lambda)}$



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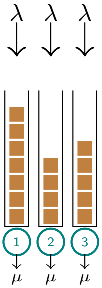

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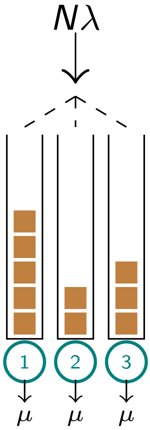
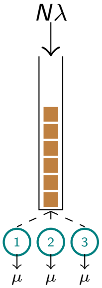


Centralized Queue

Complete Resource Pooling

M/M/N

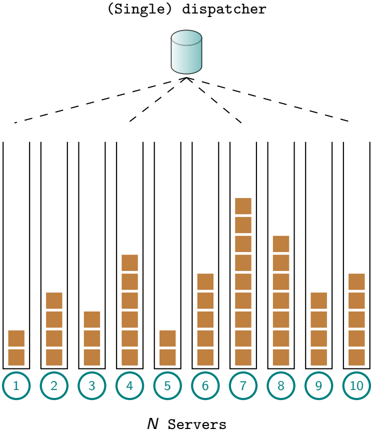


queue length $\Pi_W \frac{\lambda}{\mu-\lambda}$

waiting time $\frac{1}{N} \Pi_W \frac{1}{\mu-\lambda}$

Join the Shortest Queue (JSQ) Policy

Classical natural policy: Join the Shortest Queue (JSQ)



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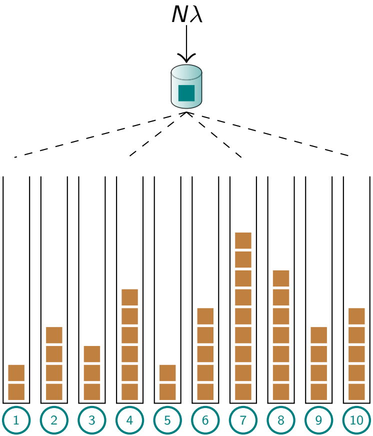

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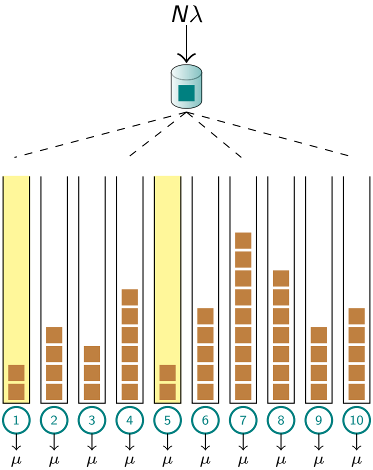

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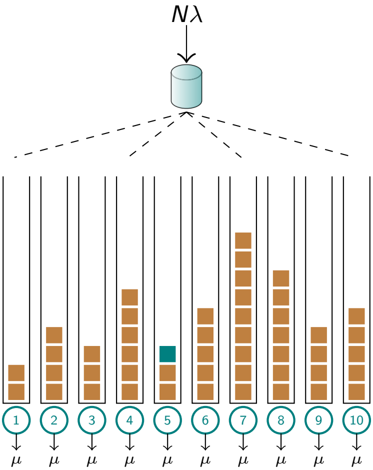

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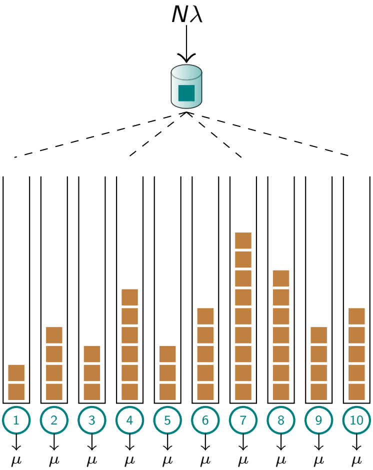

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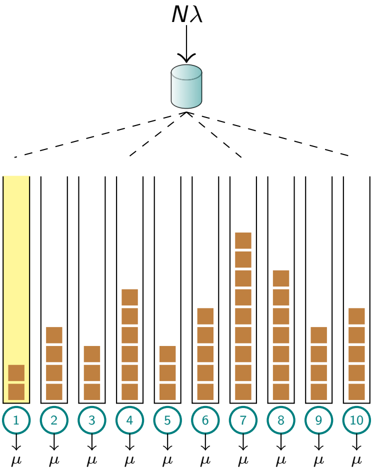

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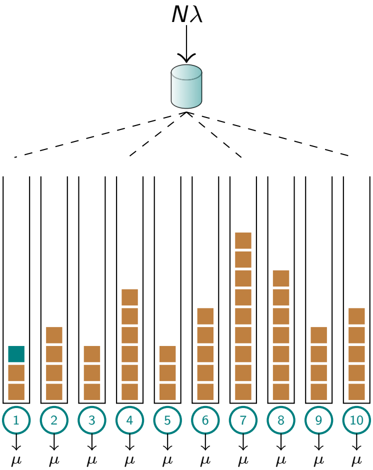

Equivalent State Description

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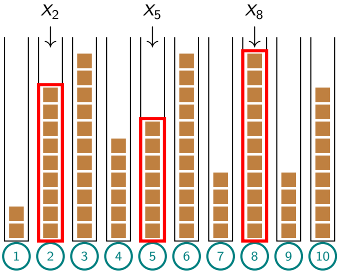

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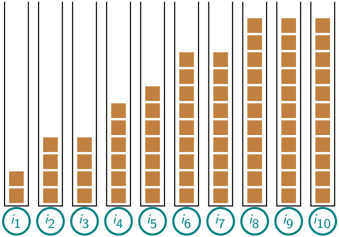

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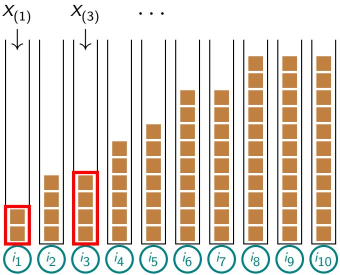

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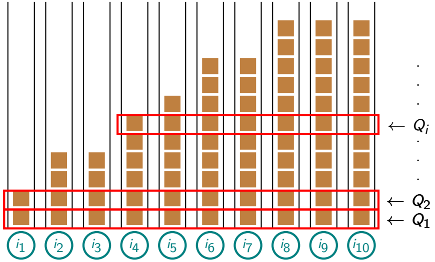

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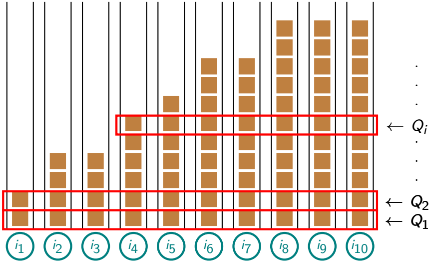
- Denote by Q_i number of servers with queue length i or larger, $i = 1, 2, \ldots$, i.e., width of i^{th} horizontal bar
- Fluid-scaled state variables $q_i^N(t) = Q_i^N(t)/N$ represent fraction of servers with queue length i or larger



Stochastic Optimality of JSQ Policy

- JSQ stochastically minimizes aggregate size of l right-most stacks, i.e., total number of tasks in l largest queues

$$\sum_{k=N-l+1}^{N} X_{(k)}^{\mathrm{JSQ}} \leq_{st} \sum_{k=N-l+1}^{N} X_{(k)}^{\Pi}$$

for all $l = 1, \ldots, N$, for any non-anticipating policy Π

[Towsley-Sparragis-Cassandras 1992; Sparaggis-Towsley-Cassandra 1994]

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- JSQ stochastically minimizes aggregate size of bars at level j or higher, i.e., total number of tasks in queue position j or higher

$$\sum_{i=j}^{\infty} Q_i^{\mathrm{JSQ}} \leq_{st} \sum_{i=j}^{\infty} Q_i^{\Pi}$$

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[Mukherjee, B, Van Leeuwaarden, Whiting 2016]

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In particular (take $l = N$ or $j = 1$), JSQ stochastically minimizes total number of tasks in system, and hence overall mean delay

JSQ Policy in Many-Server Regime

JSQ yields dramatic performance improvements as $N \to \infty$

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- Eliminates queues

Scheme	Queue length	
Random	$p_i^N = \lambda^i$	
JSQ	$p_1^N \to \lambda$, $p_2^N \to 0$	

p_i^N is stationary probability that queue length at server is $\geq i$ in N^{th} system

JSQ Policy in Many-Server Regime

JSQ yields dramatic performance improvements as $N \to \infty$

- Eliminates queues
- Achieves zero wait

Scheme	Queue length	Waiting time (fixed $\lambda < 1$)	Waiting time ($1 - \lambda \sim 1/\sqrt{N}$)	
Random	$p_i^N = \lambda^i$	$\frac{\lambda}{1-\lambda}$	$\Theta(\sqrt{N})$	
JSQ	$p_1^N \to \lambda$, $p_2^N \to 0$	$o(1)$	$\Theta(1/\sqrt{N})$	

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JSQ Policy in Many-Server Regime

JSQ yields dramatic performance improvements as $N \to \infty$

- Eliminates queues
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But...

- JSQ involves high communication overhead as $N \to \infty$
- Straightforward implementation of JSQ (no memory at dispatcher) requires queue lengths at all servers to be checked at each arrival, which may be prohibitive in large-scale systems

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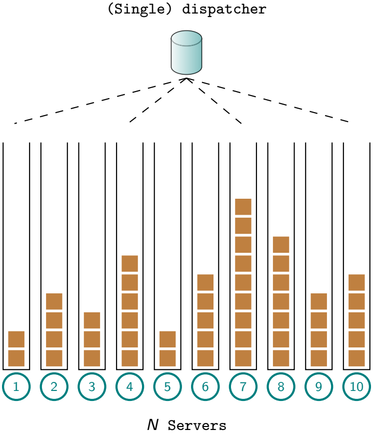
JSQ(d) Scheme: Reduced Communication Overhead

At each arrival, select d servers (e.g. $d = 2$) uniformly at random, and assign task to shortest queue among selected servers



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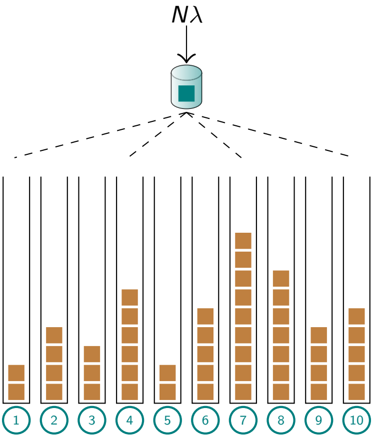

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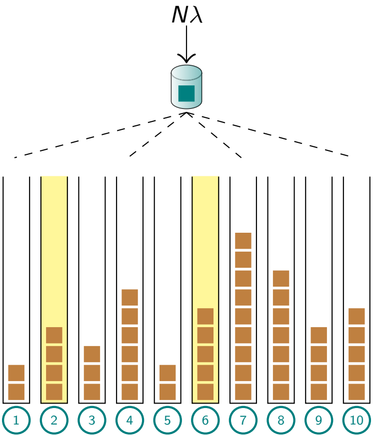

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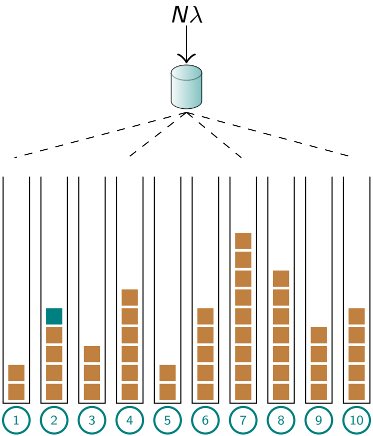

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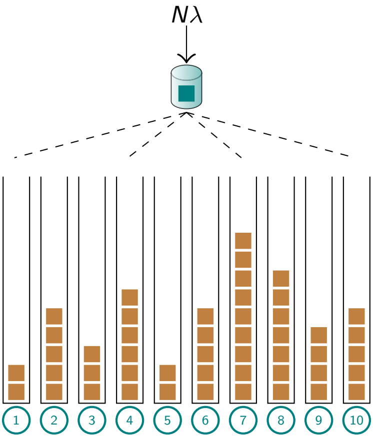

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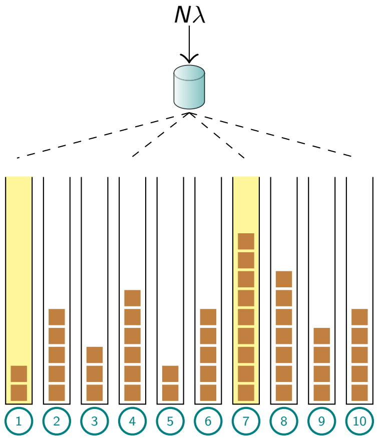

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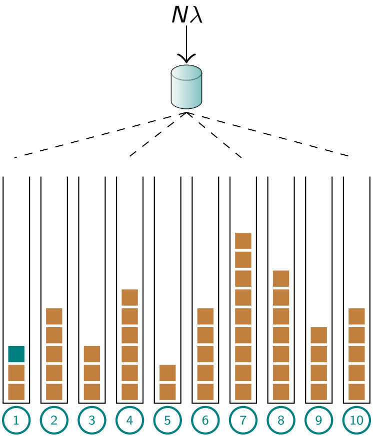

Yet Lower Communication Overhead per Task for Batch Arrivals

For batch arrivals d queue samples can be amortized over several tasks

L. Ying, R. Srikant, X. Kang (2015). The power of slightly more than one sample in randomized load balancing. In: *Proc. IEEE Infocom 2015*.

Connection between JSQ(d) and Redundancy-d Policies

Redundancy-d policy assigns replicas of each arriving task to d servers sampled uniformly at random, and kills $d-1$ replicas as soon as execution of first replica is started/completed: Wait before see rather than see before wait

K.S. Gardner, M. Harchol-Balter, A. Scheller-Wolf, M. Velednitsky, S. Zbarsky (2017). Redundancy-d: The power of d choices for redundancy. *Oper. Res.* **65 (4)**, 1078–1094.

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Fluid Limit for JSQ(d) Scheme

Fluid limit for JSQ(d) [Mitzenmacher 1996, 2001; Vvedenskaya *et al.* 1996]

If $\mathbf{q}^{JSQ(d)}(0) \to \mathbf{q}^{\infty}$ as $N \to \infty$, then $\{\mathbf{q}^{JSQ(d)}(t)\}_{t \geq 0}$ weakly converges to $\{\mathbf{q}(t)\}_{t \geq 0}$ as $N \to \infty$, with

$$\frac{dq_i(t)}{dt} = \lambda[(q_{i-1}(t))^d - (q_i(t))^d] - \mu[q_i(t) - q_{i+1}(t)],$$

where $\mathbf{q}(0) = \mathbf{q}^{\infty}$

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$[(q_{i-1}(t))^d - (q_i(t))^d]$ is (instantaneous) fraction of arriving tasks assigned to servers with queue length $i-1$ in fluid-level state $\mathbf{q}(t)$

Power-of-d Effect for JSQ(d) Scheme

Assuming $\mu = 1$ as before, fixed point of fluid limit is

$$q_i^* := \lim_{t \to \infty} q_i(t) = \lambda^{\frac{d^i - 1}{d-1}}, \qquad i = 1, 2, \ldots$$

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Tail of stationary queue length distribution falls off much faster for $d \geq 2$ than for purely random assignment ($d = 1$)

"power-of-two" effect: Even value as small as $d = 2$ yields significant performance improvements over purely random assignment, while drastically reducing communication overhead compared to JSQ ($d = N$)

JSQ(d) Provides Strong Benefits over Random Assignment

JSQ(d) provides doubly-exponential rather than (singly-)exponential decay of stationary queue length tail

Scheme	Queue length		Overhead
Random	$p_i^N = \lambda^i$		0
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In absence of any memory at dispatcher, communication overhead must grow with N in order for zero delay to be achievable [Gamarnik-Tsitsiklis-Zubeldia 2016]

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Performance versus Communication Overhead at a Glance

Random routing

$p_i^* = \lambda^i$

Performance versus Communication Overhead at a Glance

$p_i^* = \lambda^i$ $\quad$ $p_i^* = \lambda^{\frac{2^i - 1}{2-1}}$ $\quad$ $p_i^* = \lambda^{\frac{3^i - 1}{3-1}}$

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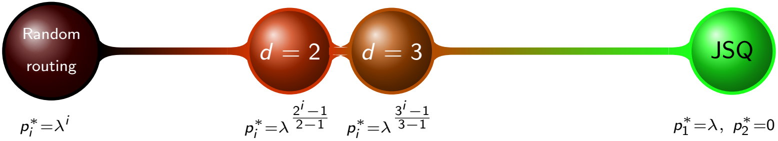

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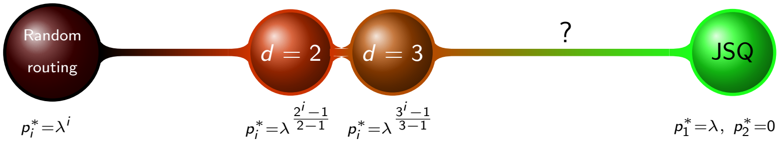

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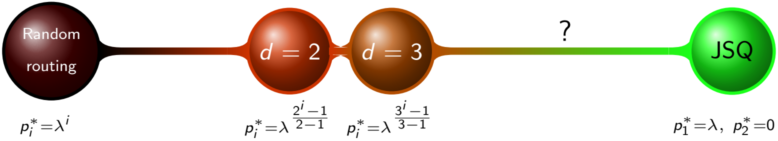
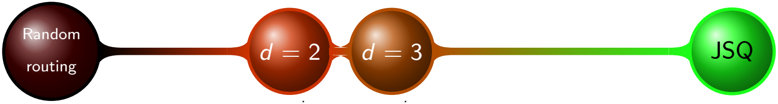


In order to examine performance versus communication trade-off, we allow parameter d to depend on N and write $d(N)$ to reflect that



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Universality and Asymptotic Optimality Properties

- JSQ($d(N)$) has same fluid limit as JSQ, and achieves fluid-level optimality when

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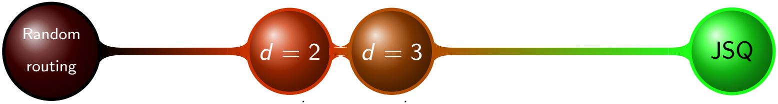

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Universality of Fluid Limit for JSQ($d(N)$) Scheme

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If $\mathbf{q}^{d(N)}(0) \to \mathbf{q}^\infty$ and $d(N) \to \infty$ as $N \to \infty$, then $\{\mathbf{q}^{d(N)}(t)\}_{t\geq 0}$ has same weak limit $\{\mathbf{q}(t)\}_{t\geq 0}$ as $N \to \infty$ as ordinary JSQ policy, with

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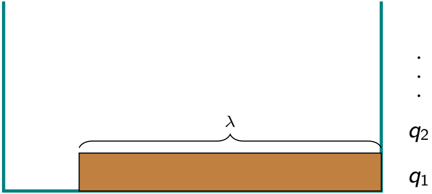
- Fixed point of fluid limit

$$q_1^\star = \lambda, \qquad q_i^\star = 0, \qquad i \geq 2$$



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$$\bar{Q}_i^{d(N)}(t) = \begin{cases} \dfrac{Q_i^{d(N)-N}(t)}{\sqrt{N}} \text{ for } i = 1 \\[2ex] \dfrac{Q_i^{d(N)}(t)}{\sqrt{N}} \text{ for } i \geq 2 \end{cases}$$

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Universality of diffusion limit for JSQ($d(N)$) scheme [Mukherjee, B, Van L, W 2016]

Assume $d(N)/(\sqrt{N}\log(N)) \to \infty$ as $N \to \infty$. Then, under suitable initial conditions, $\{\bar{\mathbf{Q}}^{d(N)}(t)\}_{t\geq 0}$ has same weak limit $\{\bar{\mathbf{Q}}(t)\}_{t\geq 0}$ as $N \to \infty$ as ordinary JSQ policy as established by Eschenfeldt & Gamarnik 2015

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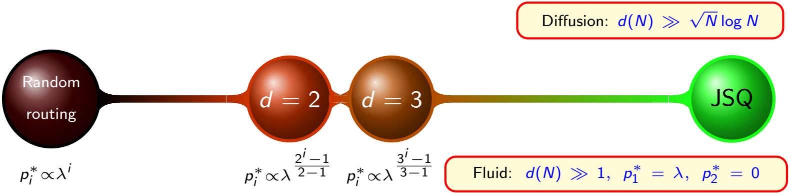
- Latter condition is nearly necessary: if $d(N)/(\sqrt{N}\log(N)) \to 0$ as $N \to \infty$, then diffusion limit of JSQ($d(N)$) scheme differs from that of JSQ policy

High-Level Summary



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JSQ(d)	$p_i^* = \lambda^{\frac{d^i-1}{d-1}}$	$\Theta(1)$	$\Omega(\log N)$	$2d$
$d(N) \to \infty$	same as JSQ	same as JSQ	??	$2d(N)$
$\frac{d(N)}{\sqrt{N}\log(N)} \to \infty$	same as JSQ	same as JSQ	same as JSQ	$2d(N)$
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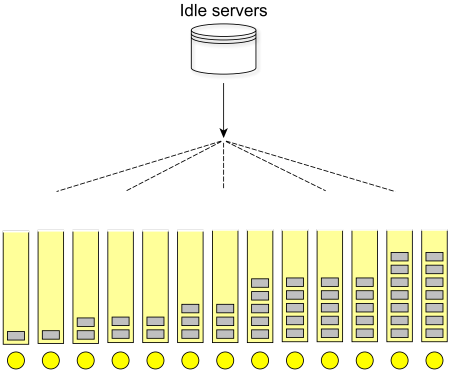
Join the Idle Queue (JIQ) Strategy: Yet Lower Overhead

Server-driven rather than dispatcher-driven [Badonell & Burgess 2008, Lu *et al.* 2011]



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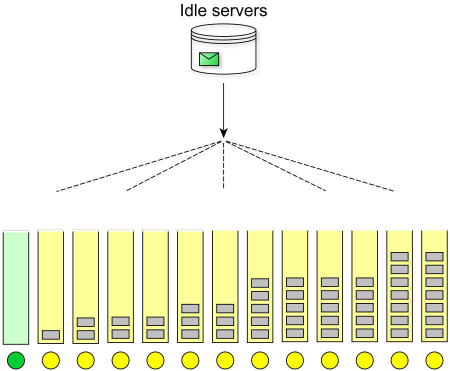

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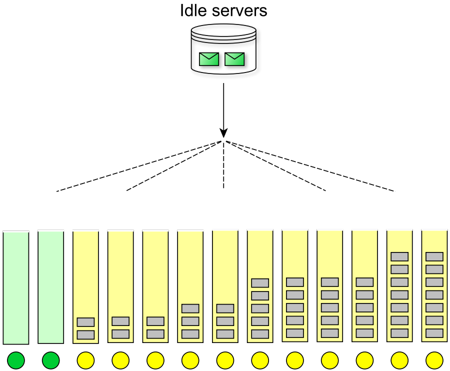

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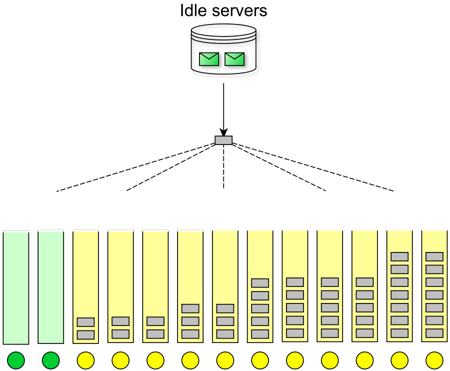

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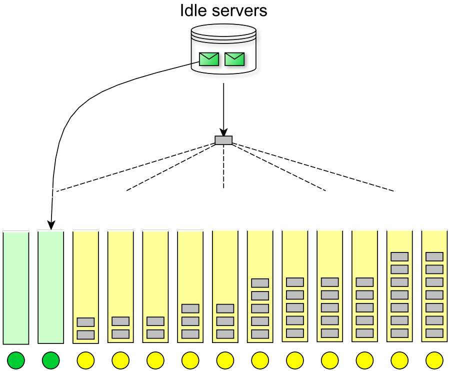

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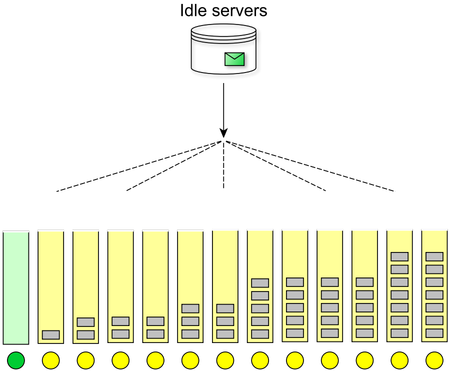

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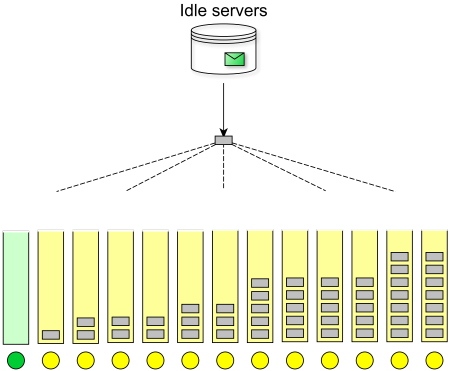

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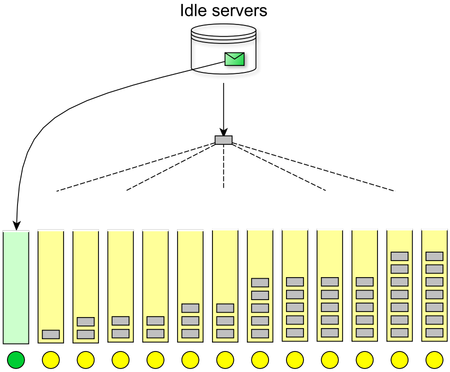

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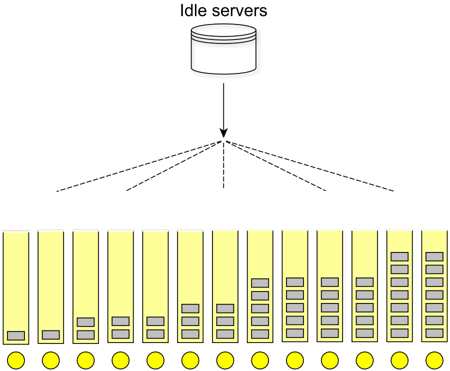

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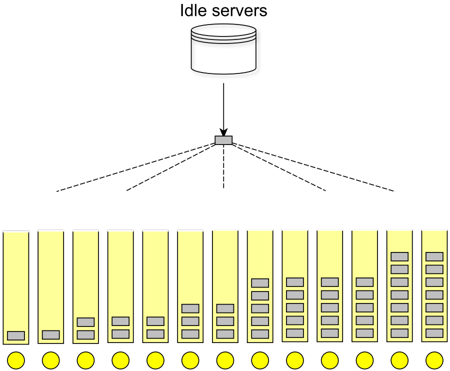

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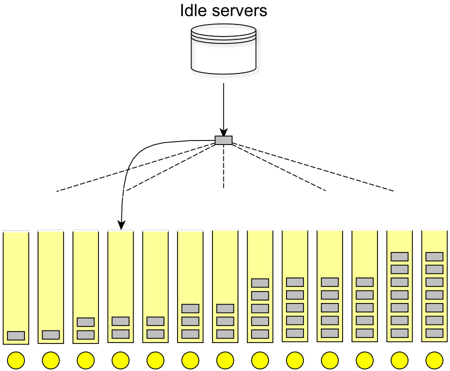

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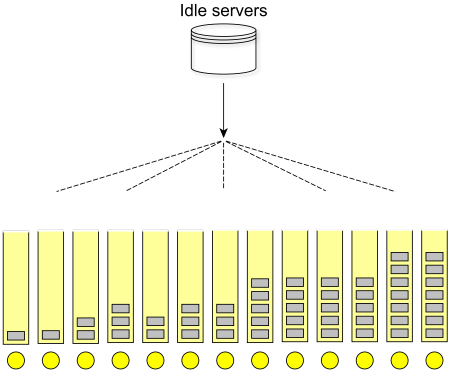

Server only sends token when service completion leaves its queue empty, implying that at most one token is generated per task

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Fixed point of fluid limit for JIQ strategy [Stolyar 2015]

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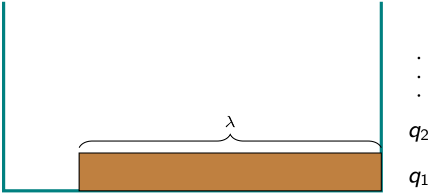

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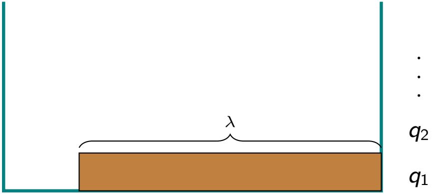

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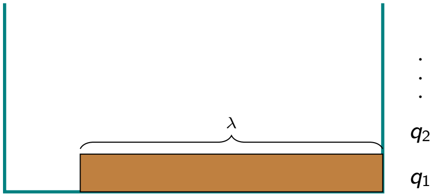

Observations

- Fluid-level behavior of JIQ strategy coincides with that of JSQ policy
- JIQ strategy achieves fluid-level optimality with O(1) communication overhead per task

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- Note that JIQ strategy uses $O(N)$ amount of memory at dispatcher
- In order for zero delay to be achievable [Gamarnik-Tsitsiklis-Zubeldia 2016]
  - either overhead per task $\to \infty$ as $N \to \infty$, so overhead per time unit $\gg N$
  - or amount of memory at dispatcher $\to \infty$ as $N \to \infty$

Performance versus Communication Overhead Trade-off

Scheme	Queue length	Waiting time (fixed $\lambda < 1$)	Waiting time ($1 - \lambda \sim 1/\sqrt{N}$)	Overhead
Random	$p_i^N = \lambda^i$	$\frac{\lambda}{1-\lambda}$	$\Theta(\sqrt{N})$	0
JSQ(d)	$p_i^* = \lambda^{\frac{d^i-1}{d-1}}$	$\Theta(1)$	$\Omega(\log N)$	$2d$
$d(N) \to \infty$	same as JSQ	same as JSQ	??	$2d(N)$
$\frac{d(N)}{\sqrt{N}\log(N)} \to \infty$	same as JSQ	same as JSQ	same as JSQ	$2d(N)$
JSQ	$p_1^* = \lambda$, $p_2^* = 0$	o(1)	$\Theta(1/\sqrt{N})$	$2N$
JIQ	same as JSQ	same as JSQ	same as JSQ	≤ 1

Extensions

- Multiple dispatchers
- Joint auto-scaling and load balancing:
  TABS scheme simultaneously achieves zero delay and zero relative energy wastage in many-server regime

Hyper-Scalable JSQ

Each server provides periodic queue status updates to dispatcher [Van der Boor, B, Van Leeuwaarden 2018]

- Either in synchronized or asynchronous manner
- At either exponentially distributed or constant time intervals with (mean) duration T and frequency $\delta = 1/T$, so overhead per task is δ/λ

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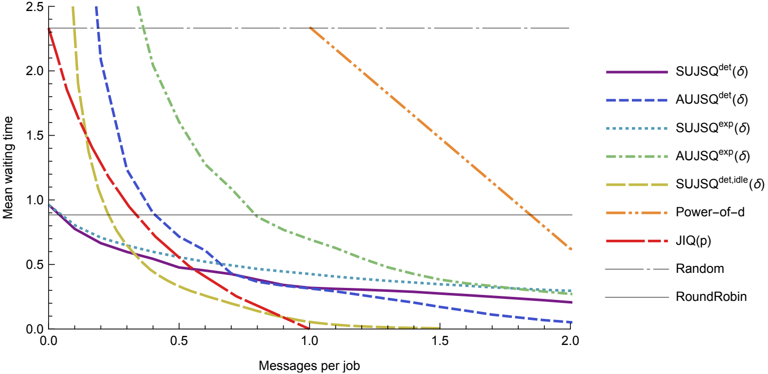
When task arrives, dispatcher assigns it server with minimum value of queue estimate (reset at updates, and incremented for each assigned task)

Hyper-Scalable JSQ



Fluid Limits for Hyper-Scalable JSQ

- $Y^N_{i,j}(t)$: number of servers with actual queue length i and queue estimate $j \geq i$ at time t

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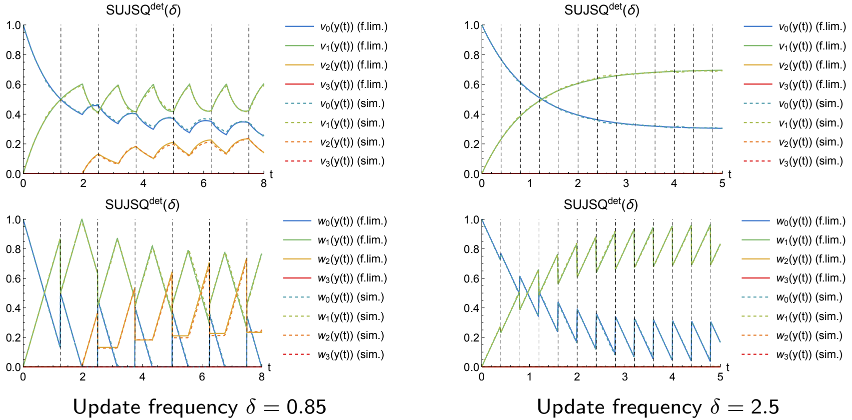
- $Y^N_{i,j}(t)$: number of servers with actual queue length i and queue estimate $j \geq i$ at time t
- $y_{i,j}(t) = \lim_{N\to\infty} Y^N_{i,j}(t)/N$: fluid-limit variables
- $v_i(t) = \sum_{j=i}^{\infty} y_{i,j}(t)$: fraction of servers with queue length i
- $w_j(t) = \sum_{i=0}^{j} y_{i,j}(t)$: fraction of servers with queue estimate j

SUJSQ^{det}: Synchronized Constant Update Intervals



SUJSQ^{det}: Synchronized Constant Update Intervals

Zero-delay threshold SUJSQ$^{det}(\delta)$

If update frequency $\delta > \frac{\lambda}{1-\lambda}$, i.e., at least $\frac{1}{1-\lambda}$ messages per task, then no queues and zero delay in stationarity at fluid level

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Finite maximum queue length $\text{SUJSQ}^{det}(1/T)$

For any $\lambda < 1$, maximum queue length in stationarity at fluid level is bounded from above by

$$M(\lambda, T) = \max\{L : \lambda T \geq H(L; \lambda, T)\} < \infty,$$

with

$$H(L; \lambda, T) = \left(1 - \frac{\lambda T}{L}\right) \left[\sum_{l=0}^{L} l \mathrm{e}^{-T} \frac{T^l}{l!} + L \sum_{l=L+1}^{\infty} \mathrm{e}^{-T} \frac{T^l}{l!}\right]$$

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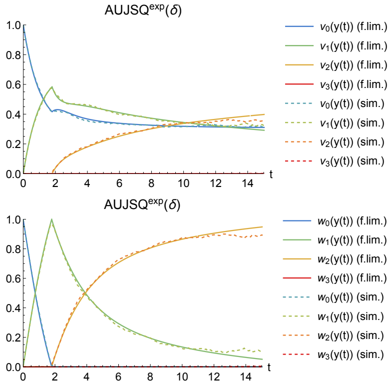
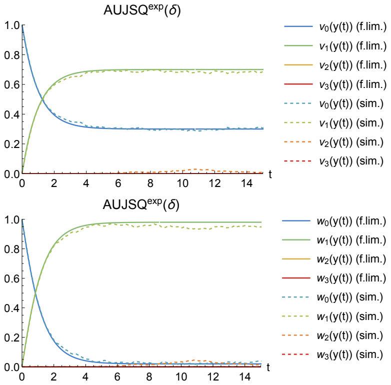
- SUJSQ$^{\mathrm{det}}(1/T)$ reduces to Round Robin policy as $T \to \infty$, and each individual server behaves as D/M/1 queue

AUJSQ^{exp}: Asynchronous Exponentially Distributed Update Intervals



Update frequency $\delta = 0.85$



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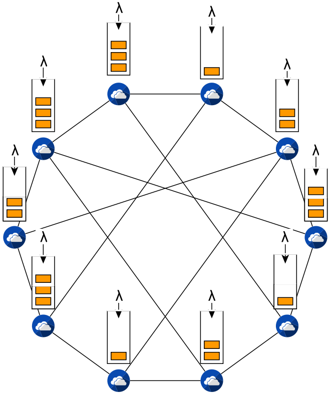
- Maximum queue length $M(\lambda, \delta) \to \infty$ as $\delta \downarrow 0$
- Queue length at individual server under SUJSQ$^{\mathrm{det}}(1/T)$ wildly oscillates as $\delta \downarrow 0$,
  rising to $M(\lambda, \delta)$ shortly after each update and then gradually dropping to low value before next update

I. Scalability challenges and classical results
II. Asymptotic optimality and universality (no memory)
III. Reduction in communication overhead (memory)
IV. Heterogeneity issues and network scenarios

Load Balancing in Network Scenarios

- Suppose N servers are interconnected by underlying graph topology G_N



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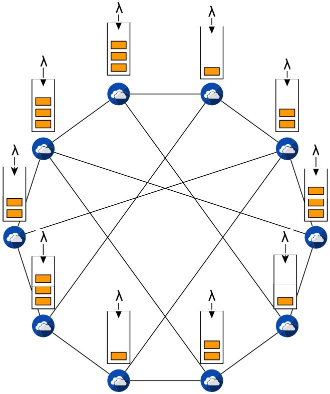

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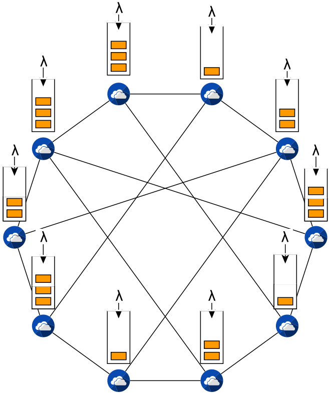

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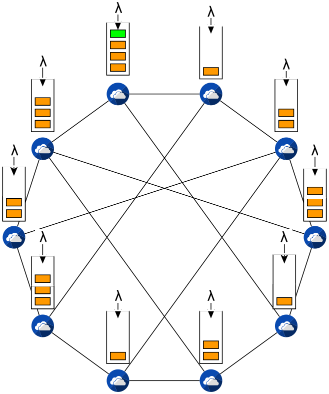

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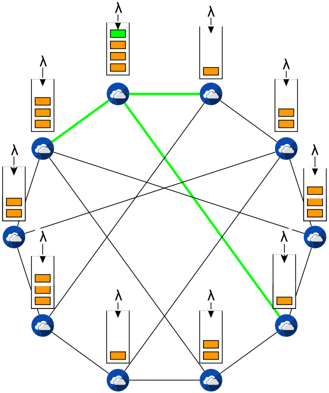

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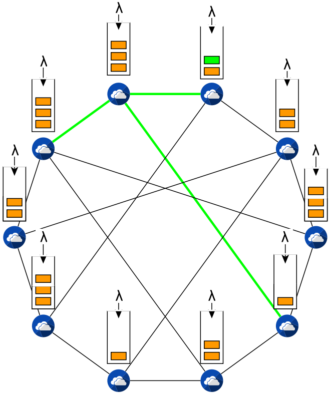

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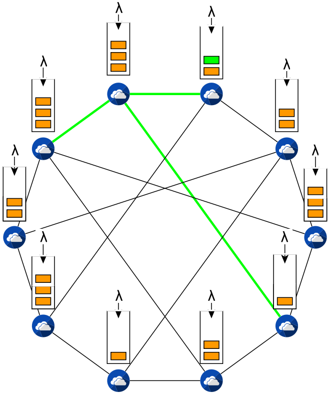

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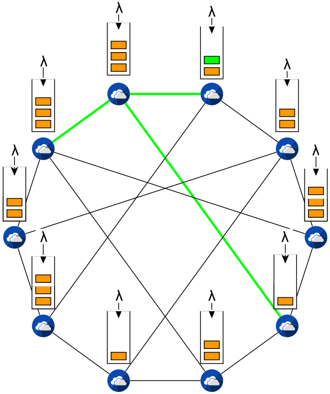
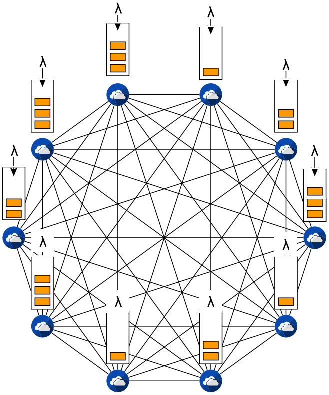
- How does graph structure affect performance?
- Lack of exchangeability among servers breaks underpinning for stochastic coupling and fluid and diffusion limits



Case of complete graph (clique) corresponds to ordinary supermarket model



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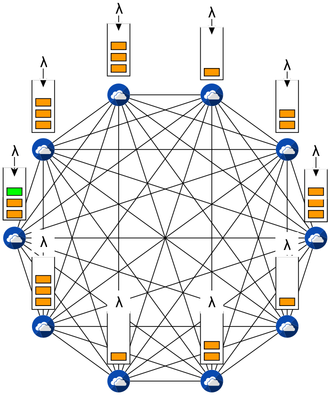

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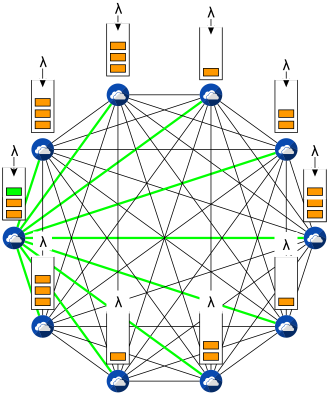

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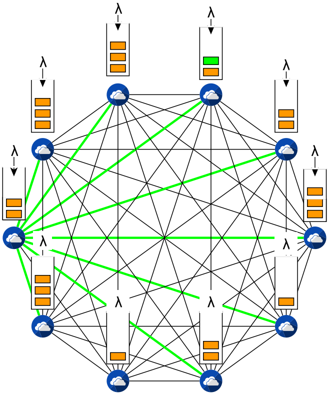
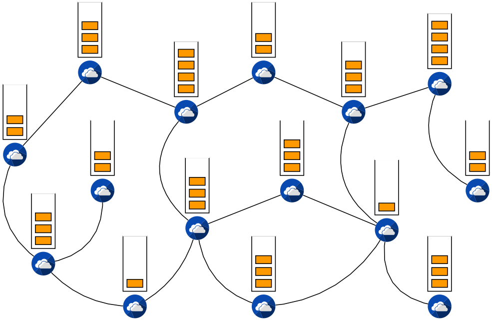


Our Perspective: Asymptotic Optimality and Universality

How much connectivity is required in order for JSQ to achieve asymptotically similar performance in G_N as in complete graph as $N \to \infty$?



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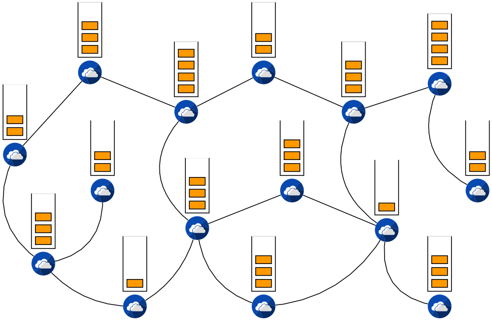
Slightly different criterion: what degree of connectivity is required for JSQ(d) to yield asymptotically similar performance in G_N as in complete graph



JSQ on Deterministic Graphs: Informal Statements

Let $\{G_N\}_{N\geq 1}$ be sequence of graphs

Condition 1

Neighborhood size of any $\Theta(N)$ vertices is $N - o(N)$

Condition 2

Neighborhood size of any $\Theta(\sqrt{N})$ vertices is $N - o(\sqrt{N})$

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Theorem JSQ on deterministic graphs

Graph sequence $\{G_N\}_{N\geq 1}$ is

(i) fluid-optimal if Condition 1 is satisfied

(ii) diffusion-optimal if Condition 2 is satisfied

Theorem JSQ on random graphs

Sequence of (Erdős-Rényi or random regular) graphs with avg degree $c(N)$ is

- fluid-optimal if $c(N) \to \infty$
- diffusion-optimal if $c(N)/(\sqrt{N} \log N) \to \infty$

Theorem Worst-case scenario

For any graph sequence $\{G_N\}_{N\geq 1}$, if

- $d_{\min}(G_N)/N \to 1$, then sequence must be fluid-optimal
- $d_{\min}(G_N)/N \to c < 1/2$, then sequence may not be fluid-optimal
- $\#$ bounded degree vertices is $\Theta(N)$, then sequence is not fluid-optimal

Conclusion

- Much sparser graphs can asymptotically match optimal performance of complete graph, provided they are suitably random
- In worst-case scenario, performance can be sub-optimal even when graph is sufficiently dense

Some References

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Thank You for your Attention!!