The generalized split probe problem

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$\mathcal{C}$ probe graphs

Given a graph class $\mathcal{C}$, a graph $G = (V, E)$ is a $\mathcal{C}$ probe if

V can be partitioned into two sets: probes P and non-probes N, such that

N is independent, and

new edges may be added between non-probes N such that the resulting graph is in the class $\mathcal{C}$

In this case, we say that (N, P) is a $\mathcal{C}$ probe partition for G

Unpartitioned $\times$ partitioned probe problems

$\mathcal{C}$ unpartitioned probe problem
instance: graph G
question: is G a $\mathcal{C}$ probe graph?

$\mathcal{C}$ partitioned probe problem
instance: graph G and vertex partition (N, P)
question: is (N, P) a $\mathcal{C}$ probe partition for G ?

Looking for separating problems

For graph classes chordal, cographs, split, threshold
both partitioned and unpartitioned probe problems are polynomial

The complement $\overline{G} = (V, F)$ of a graph $G = (V, E)$: $e \in E$ iff $e \notin F$

The complement $\overline{C}$ of a graph class C: $G \in C$ iff $\overline{G} \in \overline{C}$

Most studied probe graph classes are self-complementary!

M. Chang, L. Hung, P. Rossmanith,
Recognition of probe distance-hereditary graphs, Discrete App. Math. 2013

D. Bayer, V.B. Le, H.N. de Ridder,
Probe threshold and probe trivially perfect graphs, Theoret. Comput. Sci. 2009

M. Chang, T. Kloks, D. Kratsch, J. Liu, S. Peng,
On the recognition of probe graphs of some self-complementary classes of perfect graphs, COCOON 2005

Two conjectures

Strong probe graph conjecture

$\mathcal{C}$ probe graphs are polynomially recognizable
whenever $\mathcal{C}$ is polynomially recognizable

Probe graph conjecture

$\mathcal{C}$ partitioned probe graphs are polynomially recognizable
whenever $\mathcal{C}$ is polynomially recognizable

V. Le, H. Ridder,
Characterisations and linear-time recognition of probe cographs, WG 2007

Our contribution

Both conjectures are not true:
there exists a graph class C for which recognition is polynomial,
but both partitioned and unpartitioned probe problems are NP-complete

for (2,2) graphs,
both partitioned and unpartitioned probe problems are NP-complete

(2,1) partitioned probe problem is polynomial

(2,1) unpartitioned probe problem is NP-complete

Generalized split graphs

A generalized split (k, l) partition is
a vertex set partition into at most k independent sets and l cliques

$(2{,}0) =$ bipartite, $(1{,}1) =$ split

Full complexity dichotomy into polynomial time and NP-complete:
NP-complete if $k \geqslant 3$ or $l \geqslant 3$, polynomial otherwise

A. Brandstadt,
Partitions of graphs into one or two independent sets and cliques, Discrete Math. 1996

Generalized split partitioned probe problems

(k, l) partitioned probe problem

instance: vertex set V, edge set E, partition (N, P) of V,
where N is an independent set

question: is there a graph $G' = (V, E')$ such that
$E \subseteq E'$, all edges of $E' \setminus E$ have both endpoints in N,
and G' is a (k, l) graph?

Equivalent question:
Is (N, P) a (k, l) probe partition for G ?

Full complexity dichotomy into polynomial time and NP-complete:
NP-complete if $k^2 + l^2 \geqslant 8$, polynomial otherwise

Generalized split partitioned probe problems

Polynomial for both (2, 1) graphs and its complementary class (1, 2)

NP-complete for self-complementary class of (2, 2) graphs

(2,2) is the first known class for which
recognition is polynomial but partitioned probe is NP-complete

This shows the PGC conjecture of Le and Ridder in WG 2007 is not true

M.C. Golumbic, H. Kaplan, R. Shamir,
Graph sandwich problems, J. Algorithms 1995

R.B. Teixeira, S. Dantas, L. Faria, C.M.H. de Figueiredo,
The generalized split partitioned probe problem, LAGOS 2013

Generalized split unpartitioned probe problems

(k, l) unpartitioned probe problem
instance: graph $G = (V, E)$
question: is G a (k, l) graph?

Equivalent question:
Is there a (k, l) probe partition for G?

Full complexity dichotomy into polynomial time and NP-complete:
NP-complete if $k + l \geqslant 3$, polynomial otherwise

Generalized split unpartitioned probe problems

(1,2), (2,1), and (2,2) are the first known classes for which
recognition is polynomial but unpartitioned probe is NP-complete

This shows the SPGC conjecture of Le and Ridder in WG 2007 is not true

(1,2) and (2,1) are the first known classes for which
partitioned probe is polynomial but unpartitioned probe is NP-complete

This answers a question of Chang, Hung, and Rossmanith in DAM 2013

Further questions

There may exist a graph class C for which
the C partitioned probe problem is NP-complete
whereas
the C unpartitioned probe is polynomial

Even more interesting would be a graph class C for which
recognition is NP-complete
whereas
the C unpartitioned probe is polynomial

There may exist a graph class C for which
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Possibly, by considering M-partitions that ask for external constraints besides internal constraints might provide such examples

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