REGRA DO PRODUTO

$\rightsquigarrow D(uv) = uD(v) + vD(u) \quad \rightsquigarrow \quad \int D(uv) = \int uD(v) + \int vD(u)$

$\Rightarrow$ INTEGRAL POR PARTES : $\quad \int uD(v) = uv - \int vD(u)$

Análogo:

$$\boxed{\Delta\big(u(x)v(x)\big)} = u(x+1)v(x+1) - u(x)v(x)$$

$$= u(x+1)v(x+1) - u(x)v(x+1) + u(x)v(x+1) - u(x)v(x)$$

$$= v(x+1)\big(u(x+1) - u(x)\big) + u(x)\big(v(x+1) - v(x)\big)$$

$$= v(x+1)\,\Delta\big(u(x)\big) + u(x)\,\Delta\big(v(x)\big)$$

DEFINIR O OPERADOR DESLOCAMENTO :

$$Ef(x) = f(x+1) \quad \Rightarrow \quad \Delta(uv) = u\Delta(v) + Ev\,\Delta(u)$$

SOMA POR PARTES

$$\Delta(uv) = u\Delta(v) + Ev\Delta(u) \Rightarrow u\Delta v = \Delta(uv) - Ev\Delta u$$

$$\Rightarrow \sum u\Delta v = uv - \sum Ev\Delta u$$

<u>EX:</u> O ANÁLOGO DISCRETO DE $\int_a^b xe^x dx$ É $\sum_a^b x2^x \delta x$

TOME $u(x) = x$ E $\Delta v(x) = 2^x$

ENTÃO $\Delta(u(x)) = 1$, $v(x) = 2^x$ E $Ev(x) = 2^{x+1}$

LOGO $\sum x2^x \delta x = x2^x - \sum 2^{x+1} = x2^x - 2^{x+1} + C$

EX: O ANÁLOGO DISCRETO DE $\int_a^b xe^x\,dx$ É $\sum_a^b x2^x\,\delta x$

TOME $u(x) = x$ E $\Delta v(x) = 2^x$

ENTÃO $\Delta(u(x)) = 1$, $v(x) = 2^x$ E $Ev(x) = 2^{x+1}$

LOGO $\sum x2^x\,\delta x = x2^x - \sum 2^{x+1} = x2^x - 2^{x+1} + C$

$$\sum_a^b x2^x\,\delta x = x2^x\Big|_a^b - \sum_a^b 2^{x+1} = x2^x - 2^{x+1} + C\Big|_a^b$$

$$= b2^b - 2^{b+1} - a2^a + 2^{a+1}$$

$\sum_0^{20} = \sum_{0 \le x \le 19}$ $\qquad b=20 \quad a=0$ $\quad \underline{= 20\cdot 2^{20} - 2^{21} + 2}$

18874370